HARNESSING DYNAMICAL SYSTEMS FOR COMPLEX DECISION MAKING AND ANALYTICAL FRAMEWORKS

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Abstract

In the department of defense analysis, we prepare students by giving them tools to enable them to make good, rational decision based upon quantitative or qualitative information. This paper addresses a class exercise and class computer lab that uses a dynamical system on insurgency and insurgency versus police force to test hypothesis on decisions that could affect the outcome. EXCEL worksheets have been provided as templates to allow for experimentation, 'what-if' analysis on the coefficients, and consideration outside the box to determine the effect of decision on outcomes.

Introduction

In the first modeling course, the first block deals with *graphs as models* where the students use a graph or draw a curve to describe the situation. In the second block we cover discrete dynamical systems. We use the paradigm:

Future=Present + Change

in order to build the mathematical models. This paradigm has been used successfully before by Fox and others[1,2,4]. Excel has been shown to be a powerful technological tool for modeling with dynamical systems and other models[1,2,3,4]. This paper concerns examples of modeling with discrete dynamical systems.

Let's provide a quick example concerning Novocain.

Example 1.

Suppose that a dentist prescribes that Novocain for a patient and injects 500 mg into the jaw. Assume that the drug is immediately ingested into the bloodstream once taken. Also, assume that every hour the patient's body eliminates 25 per cent of the drug that is in his/her bloodstream. Suppose that the patient had 0 mg of the drug in his/her bloodstream prior to the injection. How much of the drug will be in his/her bloodstream after *n* hours?

<u>Problem Statement</u>:
Determine the relationship between the amount of drug in the bloodstream and time.

<u>Assumptions</u>:
The system can be modeled by a discrete dynamical system. The patient is of normal size and health. There are no other drugs being taken that will affect the prescribed drug. There are no internal or external factors that will affect the drug absorption rate. The patient always takes the prescribed dosage at the correct time. The shot is given at time 0.

<u>Variables</u>:
Define $a(n)$ to be the amount of drug in the bloodstream after period n, $n= 0,1,2,\ldots$ *hours.*

<u>Model:</u>
Mathematically, we say that the amount of drug in the bloodstream after *n* hours is

$$a(n+1) = a(n) - 0.25 * a(n) =$$
$$a(n+1) \qquad = 0.75\, a(n)$$

Initial dose: $a(0)=500$

Solution:

The solution that we desire the student to analyze is the numerical and graphical output of the dynamical system and we use an EXCEL spreadsheet.

We see in this model that the Novocain dissipates to zero over time. If the dentist knows how long the procedure take, then they can estimate the amount of Novocain to keep the patient above a threshold level (numbness) for the time it takes to perform the procedure.

In the lab for this problem, students are asked to alter the initial injection and alter the absorption rates and discuss the effects on the model's output.

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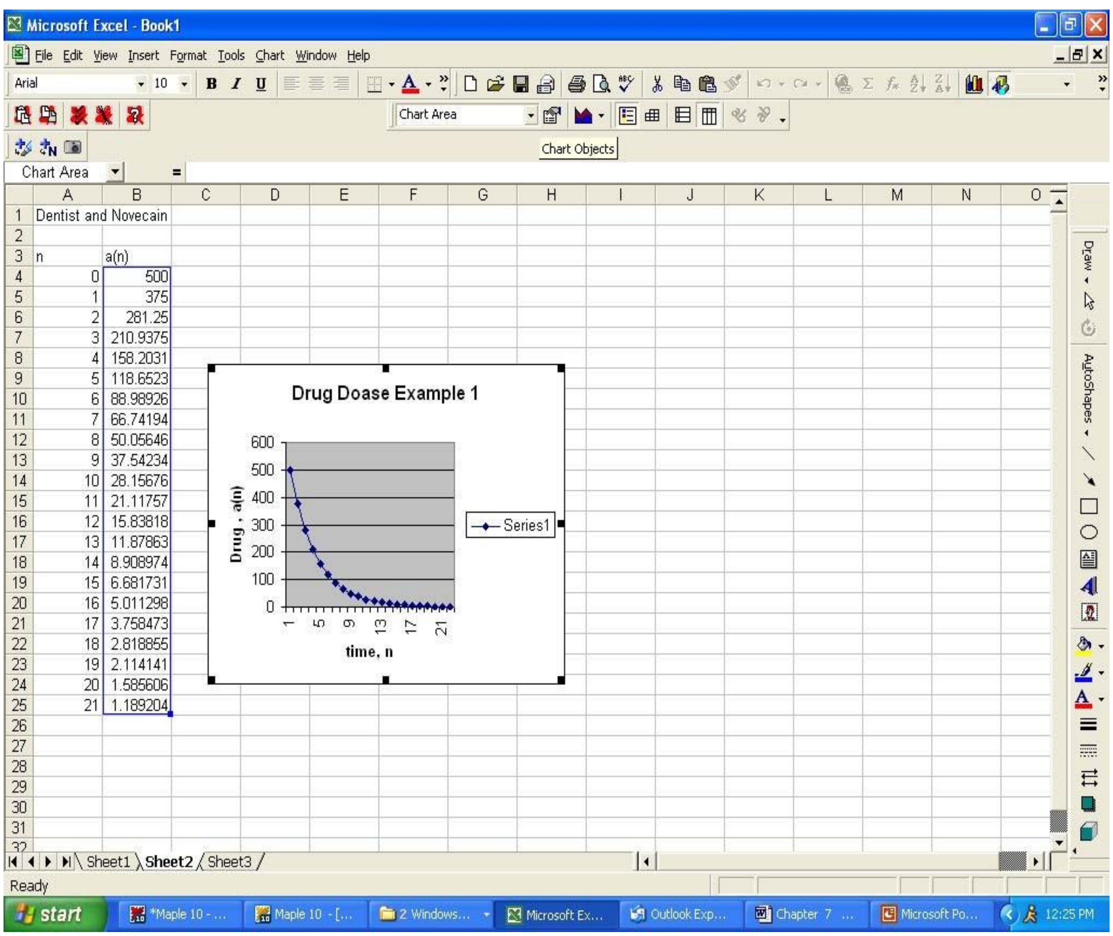
In the next problem, we change to taking a prescribed drug and have the student build the following model:

$$A(n+1)=.75\,A(n) + 200,\ A(0)=0.$$

Obtaining the numerical and the graphical solution to this model allows the students to experience two important concepts: equilibrium and stability. We provide the following useable definitions:



Equilibrium is defined when change stops, so Future=Present. At equilibrium,

$$A(n+1)=A(n).$$

Stability of equilibrium occurs when we start near and equilibrium value the dynamical system **always** iterates back to the equilibrium value.

The students are introduced to both non-linear dynamical systems and systems of dynamical systems.

For each class, we have a computer lab using EXCEL. These EXCEL spreadsheets are provided as tools for exploration and analysis by the students.

We use scenarios that build on each other in order to show the power and limitations of the computer in analysis.

Scenario 1:

Insurgent forces have a strong foothold in the city of Urbania. Intelligence estimates that they currently have a force of about 1000 fighters. Intelligence also estimates that around 120 new insurgents arrive from the neighboring country of Moronka each week. In conflicts with insurgent forces, the local police are able to capture or kill approximately 10% of the insurgent force each week on average.

Problem Statement:

Determine the relationship between the size of the insurgent force and time.

Assumptions:

The system can be modeled by a discrete dynamical system. The person is of normal size and health. There are no other factors that will affect insurgent force levels. The current force estimate occurs at time 0.

The following questions are asked of the students as they explore their model and its solution in the lab.

Questions:

1. Describe the behavior of the current system under the conditions stated:
   a.) Is there a stable equilibrium to the system under the current conditions? If so, is this an acceptable level?
   b.) How effective would an operation designed to slow (or stop) the influx of new insurgents be if the dynamics do not change?

2. What attrition rate does the police force need to achieve to drive the insurgent population to an equilibrium level below 500 in 52 weeks or less?

3. If the police force can, with advanced weapons, achieve a 30-40% attrition rate, do they also have to engage in operations to stop the inflow of new insurgents?

4. What effects do changes in the External factor, change factor, and initial condition have on the system behavior curve?

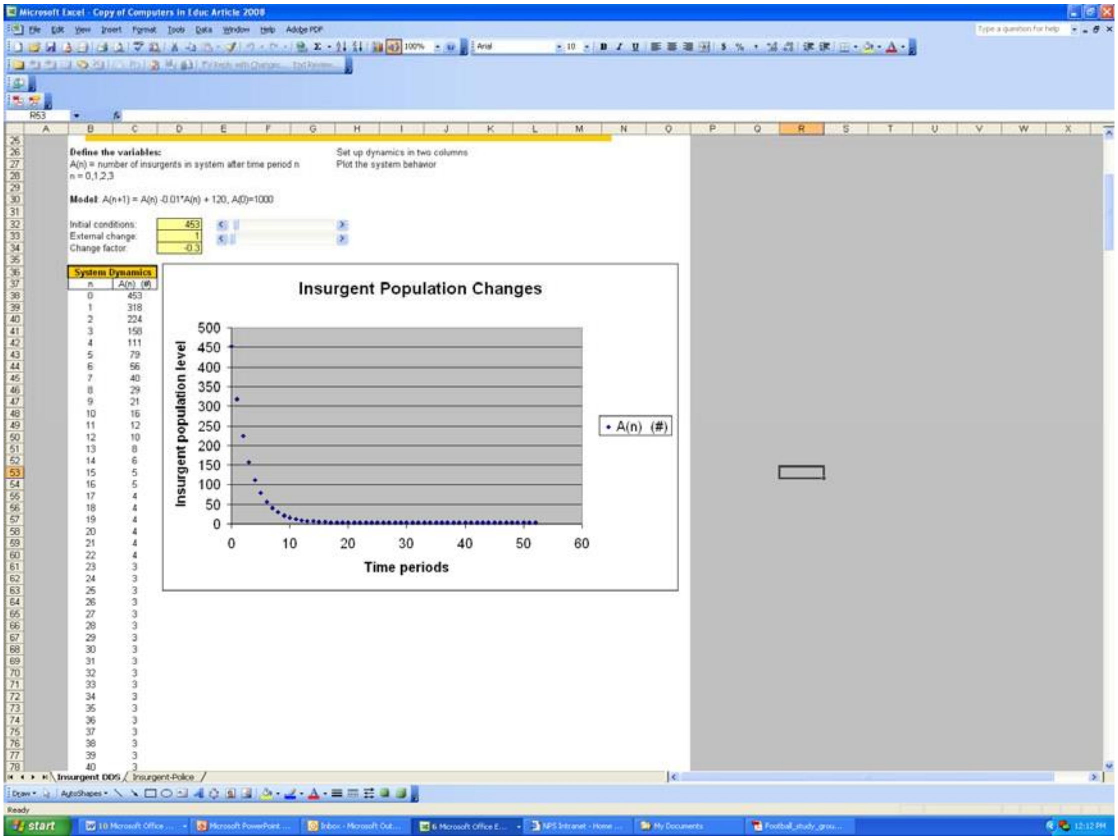
5. What conditions are necessary to cause either case (1) or (2) to occur within the 52 week horizon?

We expect the students to obtain the following model:

A(n) = number of insurgents in the system after time period, n

where n = 0,1,2,3,… weeks.

$$A(n+1) = A(n)-0.01\ A(n) + 120,\ A(0)=1000\backslash$$



Note the sliders are built into the template to allow students the ability to easily change the parameters and watch the affects on the solution dynamics.

Scenario 2:

Insurgent forces have a strong foothold in the city of Urbania, a major metropolis in the center of the country of Ibestan. Intelligence estimates they currently have a force of about 1000 fighters. The local police force has approximately 1300 officers, many of which have had no formal training in law enforcement methods or modern tactics for addressing insurgent activity. Based on data collected over the past year, approximately 8% of insurgents switch sides and join the police each week whereas about 11% of police switch sides and join the insurgents. Intelligence also estimates that around 120 new insurgents arrive from the neighboring country of Moronka each week Recruiting efforts in Ibestan yield about 85 new police recruits each week as well. In armed conflict with insurgent forces, the local police are able to capture or kill approximately 10% of the insurgent force each week on average while losing about 3% of their force.

<u>Problem Statement</u>:
Determine the equilibrium state (if it exists) for this DDS.

<u>Assumptions</u>:
The system can be modeled by a discrete dynamical system.

Questions:
1. Build the DDS.
2. Determine who wins in the long run.
3. Find reasonable values that will alter the outcome. Explain how these values could be achieved?

We define the variables

P(n)= the number of police in the system after time period *n*.

I(n)= the number of insurgents in the system after time period *n*.

n=0,1,2,3,… weeks

<u>Model</u>:

$P(n+1)=P(n)-0.03\ P(n)\ -0.11\ P(n)\ +0.08\ I(n) = 85,\ P(0)=1300$

$I(n+1) = I(n) + 0.11\ P9n)\ -0.08\ I(n)\ -0.01\ I(n) +120,\ I(0)=1000$

Again, an EXCEL template is provided to the student after they have developed the model. The template is available to provide a vehicle for analysis.

Using EXCEL

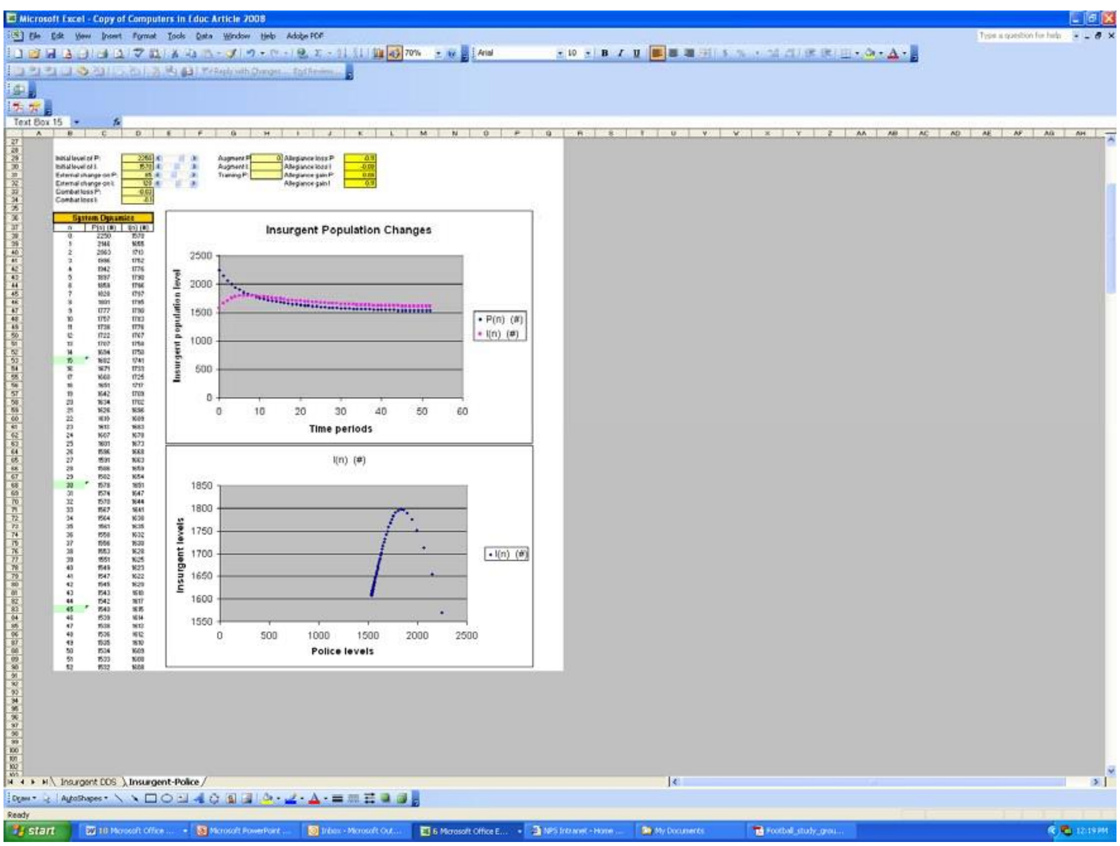
In building dynamical systems models in Excel, it is best to represent effects directly in the terms used in the model rather than simplifying the system's representation first and then building the spreadsheet model. This is because:

a. Each term and its coefficients have intuitive meaning for the problem based on a specific dynamic effect, which facilitates "what if?" analysis.
b. Exploration is facilitated more easily using Scroll Bars, which need to be linked directly to single model parameters.

Subsequent mathematical analysis, such as computing exact limiting behavior (e.g., equilibrium) follows directly by combining like terms and simplifying the system to identify the general form of the system and its solution.

Two charts are of prime interest in analysis: individual population changes via a scatter plot, and force-on-force "state space" chart. The point of this article would be rapid exploration of alternative strategies. For this one we have:

a. Complete force-on-force where both sides "see" each other and the effects noted transpire. In this setting, the two strategies are:



1) Deploy all 1500 police at the start to fight the insurgency.
2) Deploy the best 500 police at the start to fight the insurgency, and establish a formal training program that graduates 500 police every 15 weeks, augmenting the field force.

The major learning points in addition to the above would be:

a. Short term effects are represented by "periodic bump-ups" in one side or the other.
b. Long term effects are represented in the dynamics of the model, here represented by the coefficients in the system matrix.
c. Which is best to employ, separately or in combination, is difficult to determine without modeling the system behavior in a dynamic fashion.?

We do this in two examples in EXCEL: the simple insurgent model DDS in which our focus is only on tracking the insurgent population by modeling imposed effects on them only; and then introduce the two population model. In both models, we can illustrate the simplification of terms to a general form and then computing the equilibrium and examining the long term behavior.

Results and Conclusion

In Scenario 2 the police lose to the insurgents. A modification that assumes we train and graduate 500 additional police every fifteen weeks and adds them to the force improves our status but not our outcome. Students discussed that the surge in police might affect both the percentages of police and insurgents that switch sides. More insurgents will switch and less police as they all desire to be on the "winning side." Taking this into account in the model reveal that the outcome will change and the police can defeat the insurgents. This tool does not tell us how to make this happen but suggest if we can make it happen that we can alter the final outcome for the police to win, our ultimate goal in decision making assuming we are the police.

The perfect partnership of technology and modeling allows us to "test" ideas in a non-threatening atmosphere to help us make better decisions.

References

1. Fox, William P. and Rich West, "Modeling with Discrete Dynamical Systems", *Proceedings of the 16th Annual ICTCM Conference*, Addison-Wesley, pages 106-110, October 2004.

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4. West, R. D., Fox, W. P., and Fitzkee, T. L. (2007). Modeling with Discrete Dynamical Systems. Proceedings of the Eighteenth Annual International Conference on Technology in Collegiate Mathematics (pp. 101-106) Reading, MA: Addison-Wesley.

Biographical Information

William P. Fox is a professor in the department of defense analysis at the Naval Postgraduate School in Monterey, CA. He has been an educator for over 20 years, teaching at the United States Military Academy for 12 years and serving as chairman department of mathematics at Francis Marion University for 8 years prior to coming to NPS. His interests are in modeling, optimization, game theory, simulations, and statistics. He is a member of INFORMS and the Military Applications Society (MAS) of INFORMS.

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