SOLVING GRAPHIC-STATICS PROBLEMS USING AUTOCAD^{TM}

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Abstract

The graphic-statics method using a traditional drawing board was a standard procedure conceived and deployed in civil engineering education until the late 1960's supplementing analytical methods exercised to solve static forces in structural elements. However, with the advent of computers, solving such structural analysis problems has become a procedure based on computational exercises with algorithmic considerations. But solving problems of statics via graphical methods gives a vivid portrayal of the problems-in-hand conducive for visualizing the structures concurrent to the underlying calculations. Hence, the objective of this paper is to indicate how the computer-specific capabilities of AutoCAD^{TM} can be linked with the classical graphic-statics approach so as to enable civil engineering students to grasp underlying subject-matter toward rational learning of statics problems via visual graphics plus computations involved. With the use of AutoCAD^{TM} graphics software, such methods of graphic-statics (and particularly the use of so-called funicular polygon) can be a powerful and versatile approach for not only solving statics problems, but also in the analysis of civil engineering structures. Further, such efforts can be simple, fast, and carried out in a less tedious manner. Relevant graphical solutions concurrent to computer-based calculations yield visual perceptions to the problem beyond the numerical evaluation of the solutions. Combining the capabilities of AutoCAD^{TM} and the ease of visual presentations in graphic-statics forms the theme of the present paper.

Introduction

The modern student community conforms to a society of computer-literate pedagogy exposed to intense computational methods both in classroom learning as well as in the design procedures. Specific to the context of civil engineering structural designs involving static force evaluations, graphical methods were deployed classically in the conceptual stages of design with successful results. Relevant practice of this "drawing board" approach, however, dwindled slowly with computerization of the curricula. However, when structural analysis and designs are rendered entirely using computers, the "black box" of hidden computations obscures the valuable insights and visualization in conceiving the test structures being analyzed.

Though the "drawing board" era is off the classroom, the use of computer-oriented AutoCAD^{TM} has been ushered in as an alternative support in making traditional drawings of structures (with screen presentations of plan views, elevational perspectives, etc.). In addition, AutoCAD^{TM} has the computational potential to perform calculations concurrent to its drafting capabilities.

Hence, conceived in this study are methods to cash in on the graphic capabilities of AutoCAD^{TM} with its inherent structural design algorithms to revive the old graphic-statics methodology adjunct-fused with computational feasibility. This amounts to a novel curricular approach culminating as a learning tool towards understanding even intricate static problems.

Solving simple statics exercises has been indicated by Simms and Iyengar [1] by resorting to AutoCAD^{TM} graphics software. Essentially it refers to formulating a procedure using AutoCAD^{TM} graphic software to solve problems in statics. Indicated thereof are simple resultant force calculations using AutoCAD^{TM} table command with the Excel^{TM} spreadsheet.

Notwithstanding the basic educational aid proposed in [1], this paper is written to offer a more comprehensive method and a practical approach using AutoCAD^{TM} capabilities toward solving higher levels of structural engineering problems wherein the solutions are presented in a style akin to classical graphic-statics procedures. Objectively, the relevant suite of learning structured at the first level of mechanics would enable the engineering students to acquire the ability to analyze the problem in a simple and logical manner; and, they can apply to its solution a few simple and well-understood basic principles.

In this context, the use of AutoCAD^{TM} as indicated above could be elegant and simple. And, the scope of such efforts can also be expanded by fusing another convenient tool such as MatLab^{TM} in problem-solving exercises. That is, the methodology of using an AutoCAD^{TM} based approach in solving static problems can be further enhanced by teaching the students to use concurrently the potential capabilities of MatLab^{TM}. For example, in problems like those concerning trusses, the rules for static equilibrium dictate a corresponding number of independent equations that define an eventual solution; and, MatLab^{TM} is a convenient tool thereof in finding solutions on the static forces in the structures of interest. Again, such results can be used to fortify the AutoCAD^{TM} solutions. Hence, a conceivable pedagogy may judiciously include simultaneous use of AutoCAD^{TM} and MatLab^{TM} strategies that can be built on the approach advocated in this study.

The students who may use the process described here can be at sophomore level of Engineering, Technology, and Architecture programs. In order that relevant class-room efforts are to be combined with hands-on, computer-based AutoCAD^{TM} (plus MatLab^{TM}, Excel, etc,) tasks, the class-size may be confined to match the infrastructure of the facility provided. However, in the event of large classes, parallel sessions can be conceived.

The following are example problems indicated to illustrate the novelty of using the AutoCAD^{TM} in graphic-statics problem-solving:

- Basic force calculations in structures like a simple truss
- Graphical analysis of multipanel trusses
- Application of funicular-polygon principle in AutoCAD^{TM}-based solutions in graphic-statics format

Forces in a Simple Truss

Statement of Problem # 1: This refers to solving for forces in a simple truss. (A truss is a collection of members arranged in a triangle or connected triangles so that the forces that act on its joints cause only axial tensile or compressive forces in the members). Consider an example of a simple truss shown in Figure 1, where a body of mass 50 kg is supported at the top of a triangular framework of wooden members pinned together by a single bolt at each joint. Further, this framework has the shape of a 30°-60°-90° triangle with its hypotenuse placed in a horizontal orientation [2]. It is held up by reactions at its two lower corners, 3 m apart. One of its lower corners is supported by a hinge that allows free rotation; the other sits on a roller permitting free horizontal motion. This framework conforms to a simple truss. The exercise is to determine the forces in each member of the truss.

First, the force due to gravity exerted on the vertically suspended 50-kg mass is calculated as follows:

$$P = mg = (50 \text{ kg}) \times (9.8 \text{m/s}^2) = 490 \text{ N} \qquad (1)$$

Next, a free-body diagram of the truss under discussion can be drawn as illustrated in Figure 2 where R_B and R_C are the reaction components. AutoCAD^{TM} can be used to specify the geometrical lengths involved in the structure. These lengths are needed to solve for the reactions, R_B and R_C as illustrated in equations (2) and (3).

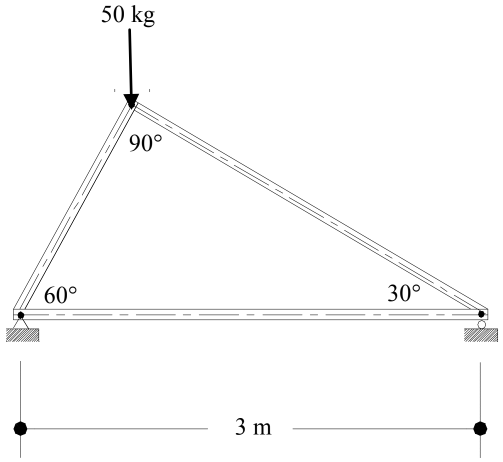

Figure 1. A Simple Truss.

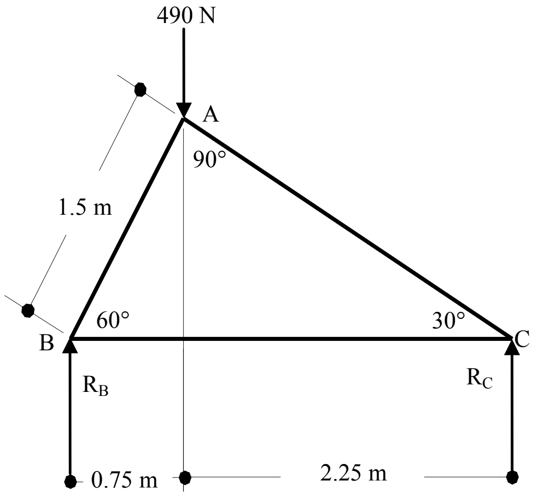

Figure 2. Free-body Diagram of the Simple Truss.

$$\sum M_B = 0 = (490\text{N})(0.75) - \text{R}_\text{C}\,(3\text{m}) \qquad (2)$$

$$\text{R}_\text{C} = 123\text{ N}$$

$$\sum \text{M}_\text{C} = 0 = -(490N)(2.25m) + R_B(3m) \qquad (3)$$

$$\text{R}_\text{B} = 367\text{ N}$$

Now, suppose a graphical analysis of the truss is attempted using AutoCAD$^{\text{TM}}$ without invoking any arithmetic, algebraic, or trigonometric computations. This can be done first by drawing an accurate geometry (triangle) of the truss with the labeling of the applied and reaction forces as illustrated in Figure 3.

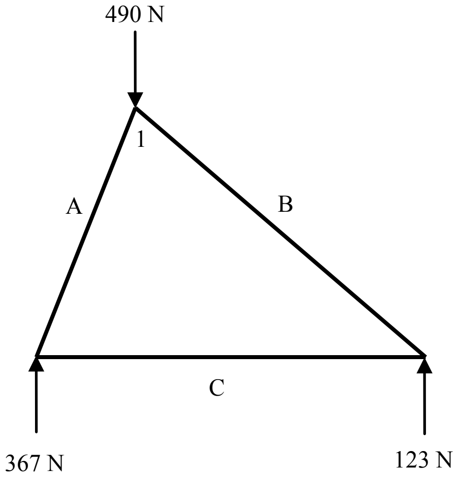

Figure 3. Form-Diagram.

Figure 3 is known as a "form-diagram" where relevant notations used are as follows: Capital letters are placed in the intervals between external forces on a truss; and, numbers are indicated in the internal spaces between members. Thus, the load 490 N is referred to as force AB. The left-hand sloping member is A1, and the top joint in the truss is indicated as AB1.

Next, a force polygon shown in Figure 4 is constructed using AutoCAD$^{\text{TM}}$. This force polygon begins with a load line to represent the vertical load of 490 N denoted as AB. The capital letters that designate intervals on the form-diagram correspond to lowercase letters that depict the points on the load-line; and, the order of the letters on the load-line denotes the directional sense of the force. For example, inasmuch as BC acts upward, this length is plotted on the load-line beginning at b and finishing as 123 N above at c, indicating its upward direction.

As a next step, the forces in the members of the truss are determined: Starting at the left reaction locale, it can be seen that two members

A1 and C1 meet there. These two forces must be in equilibrium with respect to the left reaction, CA. Correspondingly, AutoCAD^{TM} will give the lengths of a1, a line through "a" parallel to A1, and of c1 depicting a horizontal line through "c". Thus, the triangle a1c on the force- polygon is an equilibrium polygon for the forces at joint A1C of the truss totally conceived via graphic capability of the AutoCAD^{TM}.

Continuing the graphical solution as above, one can plot another triangle of forces, bc1, on the force polygon to represent the equilibrium of forces at joint BC1 of the truss (Figure 4). Again, the AutoCAD^{TM} gives the lengths as required.

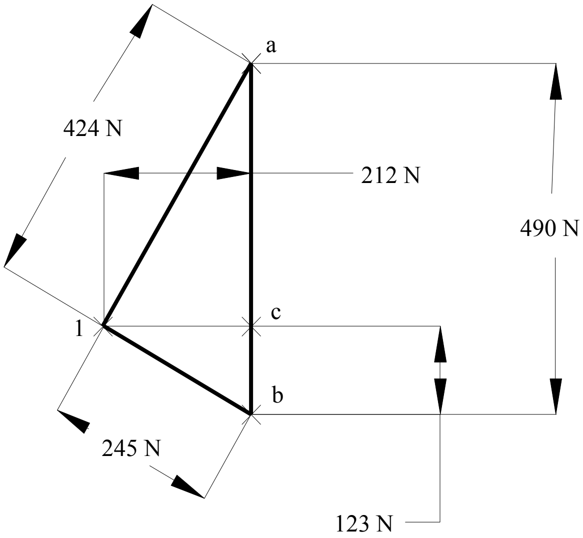

Figure 4. Force Polygon.

The summary diagram of member forces of the Problem #1 is presented in Figure 5. As can be observed, AutoCAD^{TM} proves to be very useful in performing graphical analysis of a structure like a simple truss. Without any loss of generality, such graphical analysis in deducing a force polygon can be extended even for complex trusses as described in the next example.

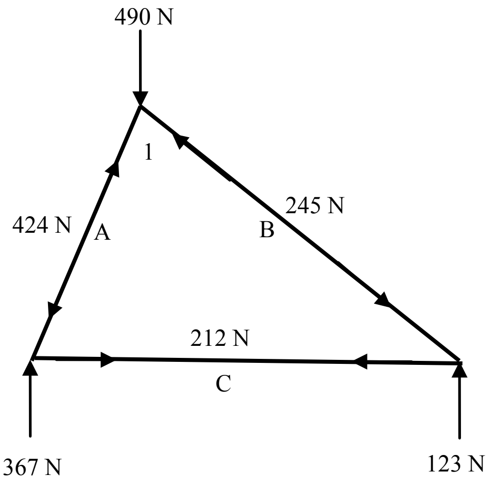

Figure 5. Summary Diagram of Member Forces in the Simple Truss of Problem #1.

Multipanel Trusses

Statement of Problem # 2: This is concerned with larger and more complex trusses using the AutoCAD^{TM}. As an example, a six-panel flat truss with 45^{o} diagonals that slope downward toward the center of the truss as illustrated in Figure 6 [2] can be considered where a vertical load of 1600 lb is applied to each panel point along the top chord, and the total span of the truss is 48 ft.

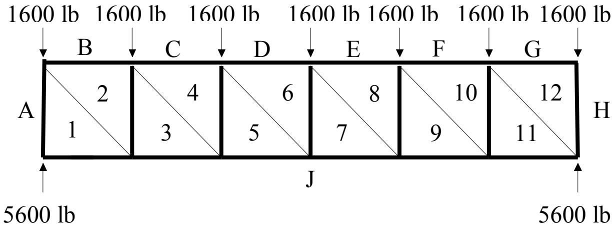

Figure 6. Form-Diagram of a Six-Panel Flat Truss.

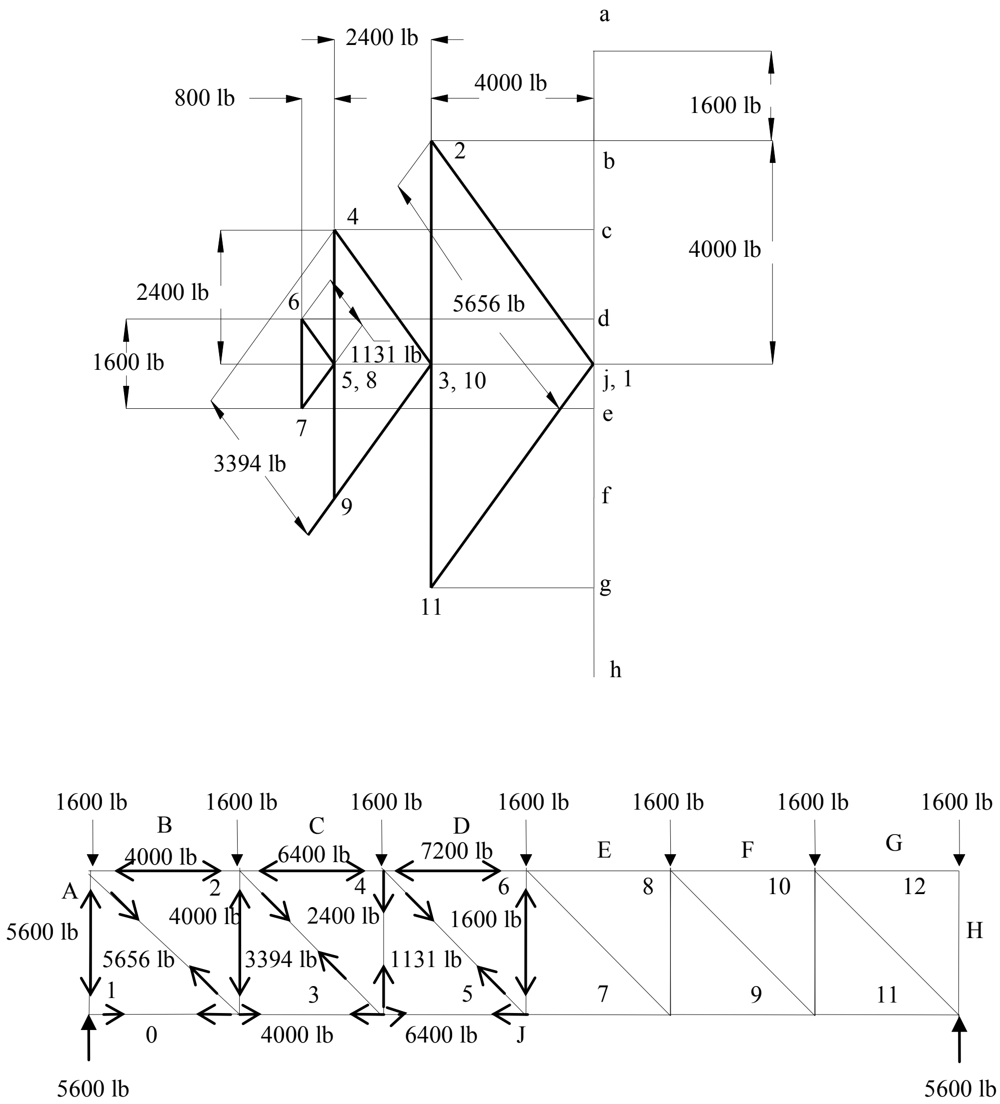

Figure 7. Force Polygon (Not-to-scale) and Adding Member Forces to the Six Panel Flat Truss.

As shown in Problem # 1, first the AutoCAD^{TM} is used to draw an accurate form-diagram of the truss, and relevant vectors are drawn to represent the loads on the truss (Figure 6). The two reactions are shown on the truss and added to the force diagram as vectors. Because the loading is symmetrical, only half of the total load to each reaction is assigned. Again capital letters are assigned to spaces between external forces and numbers to internal spaces, as specified in the previous example.

Next a force polygon is constructed by drawing a load-line using AutoCAD^{TM} procedure (Figure 7). Each increment of vertical length on the load-line denotes each of the 1600 lb vertically applied loads. This leads to tracking from "a" to "h" at the bottom. The right-hand reaction being “hj”, an upward-acting force is plotted on the load-line as a segment that begins at h and extends upward to j at a location that is 5600 lb above. The left-hand reaction being “ja”, another upward force closes the load-line back to the point of origin.

Examining the left-end of the truss, the only joint with fewer than three unknown forces is A1J; and, as such, this is where the graphical analysis can be commenced. On the force polygon, the forces in the two members that meet at A1J are represented by a vertical-line segment, a1 that intersects a horizontal-line segment j1 at point 1. The line segment a1 must pass vertically through a, which means that it lies along the load-line, and j1 must pass horizontally through j. These conditions can be satisfied only if point 1 lies precisely at j. Because it has zero length, j1 is a point rather than a line, implying that J1 is a zero-force member.

Now one can determine the forces in members B2 and 2-1. Lines b2 and 2-1 are constructed parallel to these members on the force polygon with AutoCAD™. Thus, b2 is a horizontal line through b on the load line, and 2-1 is a diagonal line through 1. The point of intersection of these lines is the location of point 2. In order to find the unknown forces in members 2-3 and 3J, lines 2-3 and 3j are plotted on the force polygon parallel to these members to find point 3. As illustrated in Figure 7, the process as above is repeated, moving from joint-to-joint across the entire truss; and, using AutoCAD™, the line segments in the force polygon are conveniently measured to find their magnitudes. Figure 7 in essence shows the resulting details obtained on half the truss. The member forces are shown for only one half of the truss for brevity.

Funicular Structures

Statement of Problem # 3: AutoCAD™ can also be used to find graphically the solution to certain problems in statics by constructing a funicular polygon, which denotes a form that a cable would assume under a given set of loading. The concept of funicular polygon is useful in analysis of hanging structural elements in a suspension bridge or cable-supported roof. In such contexts, it is necessary to predict the form that a cable would assume under a given set of loads that may act on these structures. Concurrently, the magnitudes of tensile forces that will be caused by these loads in all parts of the cable should be ascertained [2]. Relevant graphic-static analysis can be exercised using AutoCAD™.

As an example, consider a suspended cable shown in Figure 8. It supports three different concentrated loads that are applied at irregular intervals. At its left support, it runs over a frictionless pulley to a 10,000 lb weight. When the system is at rest, the left-most segment of the cable lies at an angle of 45^o to the horizontal.

AutoCAD™ can be used to find graphically the forces in each segment as well as the direction of each segment. First, the AutoCAD™ is used to construct the funicular polygon. The top portion of Figure 9 shows the loading diagram which is a device used for hanging cables and arches to facilitate a consistent application of interval notation. The horizontal projection of the cable is represented as a line whose length is equal to the span. The distances between the lines of action of the loads are shown along this line. Loads and reactions are shown as vectors. Capital letters are applied to the spaces between the loads, using the letter "O" to denote the space below the horizontal line. The funicular polygon is constructed projecting downward along the lines of action of the loads. The left-most segment of the funicular polygon is denoted oa and its direction is given as 45^o to the horizontal (Figure 9). Further, a force polygon for the first node, ABO (Figure 10) is constructed using AutoCAD™ and the vertical vector, ab, exactly 6000 lb long is plotted. The vector that represents the known direction and magnitude of the force in cable segment OA is obtained from the AutoCAD™ plot oa. As can be seen from Figure 10, the segment oa starts at point o, downward and to the right from point at a distance of 10,000 lb.

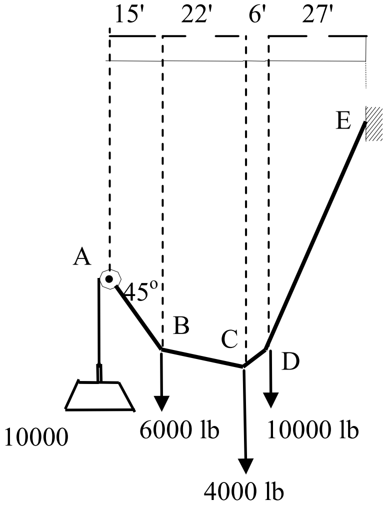

Figure 8. Hanging Cable with a Nonuniform Loading.

A vector from o to b completes the force polygon and it denotes the direction and magnitude of the force in cable segment OB. Now, the direction of vector ob can be transferred directly to the funicular polygon where it represents the direction and length of segment ob of the cable.

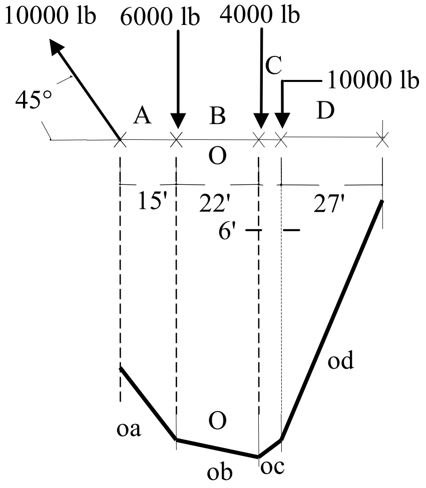

Figure 9. Funicular Polygon.

Moving to node BCO, a second triangular force polygon is plotted with AutoCAD^{TM}. Its known side namely, ob is found by completing the first triangle and bc. The third vector, oc, has a magnitude that scales to approximately 7600 lb. Next oc is drawn on the funicular polygon (Figure 9), parallel to oc on the force polygon. A third triangle, cdo (Figure 10), completes the graphical solution, giving the magnitude of the force DO and allowing the plot od on the funicular polygon (Figure 9).

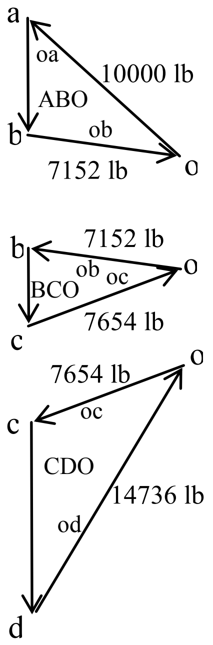

Figure 10. Individual Force Polygons.

Conclusion

The present study illustrates the use of AutoCAD^{TM} in obtaining graphic-statics solutions for basic statics problems [3] as well as in more complex structural engineering analyses. Such AutoCAD^{TM}-based graphical solutions in essence yield visual details in addition to knowing the forces and directions. Further, relevant efforts can be carried out fast and in a less tedious manner. Solving statics problems via computation only without graphical aid deprives the designer of valuable insights into the behavior of the structure being designed. Also invaluable clues on how to improve the form of the structure are lost without the graphics on the design details. On the other hand, by resorting to graphical methods, more visual aspects of structural elements and the associated force details are captured. Relevant graphic-static pursuits can be conveniently enabled using the AutoCAD^{TM} software supporting simultaneously the calculations facilitated via algorithms programmed in the computer.

The efficacy of the method described in this study is reflected in the class-room efforts pursued by the authors. The strategy of pedagogy described was adopted at the sophomore level in the Department of Civil, Environmental, and Geomatics Engineering, at FAU. The general feedback and learning experience indicated by the students has been encouraging. More exercises and design problems are being planned in the near future. Specifically, use of AutoCAD^{TM}, Excel^{TM} and MatLab^{TM} cohesively in similar analyses and design problems (involving graphic-statics) is planned. It is expected that such exposures will enhance the learning potential and fastness in obtaining solutions in addition to the advantage of visualization.

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