USING STUDENT KNOWLEDGE OF LINEAR SYSTEMS THEORY TO FACILITATE THE LEARNING OF OPTICAL ENGINEERING

Cameron H. G. Wright
Department of Electrical and Computer Engineering
University of Wyoming, WY

Thad B. Welch
Department of Electrical and Computer Engineering
Boise State University, ID

Michael G. Morrow
Department of Electrical and Computer Engineering
University of Wisconsin-Madison, WI

Abstract

For students learning a new topic, being able to use existing knowledge and mental models in the context of the new topic leads to faster learning and a deeper understanding of the new concepts. This paper describes how teaching a graduate-level course providing an introduction to optical engineering for students from multiple engineering majors can be facilitated by using existing concepts and knowledge of linear systems theory, which are common to them all.

Introduction

This paper presents an effective and efficient method of teaching a subject (optical engineering) which is new to students from various engineering disciplines. In particular, this method leverages existing student knowledge of linear systems theory to facilitate their learning of this new subject more quickly and intuitively, and makes extensive use of MATLAB plots and simulations as a primary tool. The specific challenge was the need for graduate students from various engineering disciplines to develop, in only a single course, a practical working knowledge of optical engineering to support their research efforts. That is, many research projects were relying on digital cameras and other imaging systems to obtain critical data, yet the students had no background in optical engineering. Therefore, the ability to design an appropriate imaging setup, or to know what limitations should be taken into account when interpreting image data from existing setups, was completely lacking. Without any background in optical engineering, common errors or misconceptions could result in the students' research data being tainted or even useless.

In order to optimize the learning environment for engineering students, the various theories of how best to teach adults (andragogy) can be studied and utilized [1]. One apropos andragological learning theory is that of constructionism, which builds on Piaget's highly regarded epistemological theory of constructivism [2–4]. The most salient aspect of constructionism that applies to the subject of this paper was well described by Ausubel in reference [5]:

> "If I had to reduce all educational psychology to just one principle, I would say this: The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him/her accordingly" (p. iv).

That is, how easily our students can learn a new topic depends to a large degree on their prior knowledge, or what some researchers call their existing cognitive frameworks. This comes as no surprise to most experienced professors, but it can be reassuring to know that the method has a strong foundation in educational psychology.

The Challenge

The students who needed this introduction to optical engineering included those from electrical, computer, mechanical, civil, and chemical engineering. Some had been briefly exposed to traditional first-order optics in a sophomore physics class, but most had not. However, all the students had a background in linear systems theory that included convolution, impulse response, transfer function, Fourier transforms, and so forth, whether applied to electrical circuits, mechanical systems, or chemical processes. They also had experience with MATLAB to varying degrees. That was an existing cognitive framework that could be leveraged for more effective learning.

The authors have many years of experience taking advantage of existing cognitive frameworks, active learning methods, and experiential exercises using both MATLAB and C to teach digital signal processing topics to students whose background was often just a Signals and Systems course [6–12]. Signals and Systems is the electrical and computer engineering focused version of linear systems theory. Leveraging more general linear systems theory to teach a much broader audience of students the topic of optical engineering was a new challenge.

The Method

The key, as described above, is to make the best use of what the learner already knows. Rather than taking the more traditional (and lengthy) route of first teaching theoretical optics (first-order ray tracing, interference, Rayleigh-Sommerfeld diffraction, Fresnel diffraction, Fraunhofer diffraction, third-order aberrations, higher-order aberrations, etc.) then following that with application-oriented optical engineering, we chose to go directly to practical optical engineering by way of linear systems theory and what is often called Fourier optics [13]. That is, we quickly established the link that what they already knew as an impulse response is really just the equivalent, in the optical engineering context, of a point spread function (PSF). The students also already knew that the Fourier transform of the impulse response is the system transfer function (from which the frequency and phase response are derived), and so we could easily establish the link that the Fourier transform of the PSF is the optical transfer function (OTF). The magnitude of the OTF is the very important modulation transfer function (MTF), which is heavily used in both the analysis and the design aspects of modern optical engineering, and can be thought of as the "frequency response" of the optical system. When dealing with lenses and lens systems, the Fourier transform is continuous, and when dealing with typical sampled sensor arrays (such as those used in essentially all digital cameras), the Fourier transform is discrete [13–16]. However, both are typically calculated by using the always-discrete FFT using a modern engineering tool such as MATLAB.

As an example of how MATLAB is used in this way, the Fourier transform pair relationship between the PSF and MTF is shown in Fig. 1. The top part of the figure depicts the diffraction-limited Airy disk due to a circularly symmetric lens and aperture arrangement, with the common assumptions of zero aberrations and far-field conditions. The bottom part of the figure is the MTF of the same optical system. From this sort of analysis, using little more than linear systems theory as applied to optics, students can quickly begin to see that there is a "cutoff frequency" predicted by the MTF. No spatial frequency higher than D/λ can be imaged by this optical system, and no valid conclusions may be drawn from image data when assumptions exceed this physical limit. Another observation from this simple example is that the best possible contrast of details in an image decreases as the spatial frequency increases, since contrast is a function of the MTF value for a given spatial frequency.

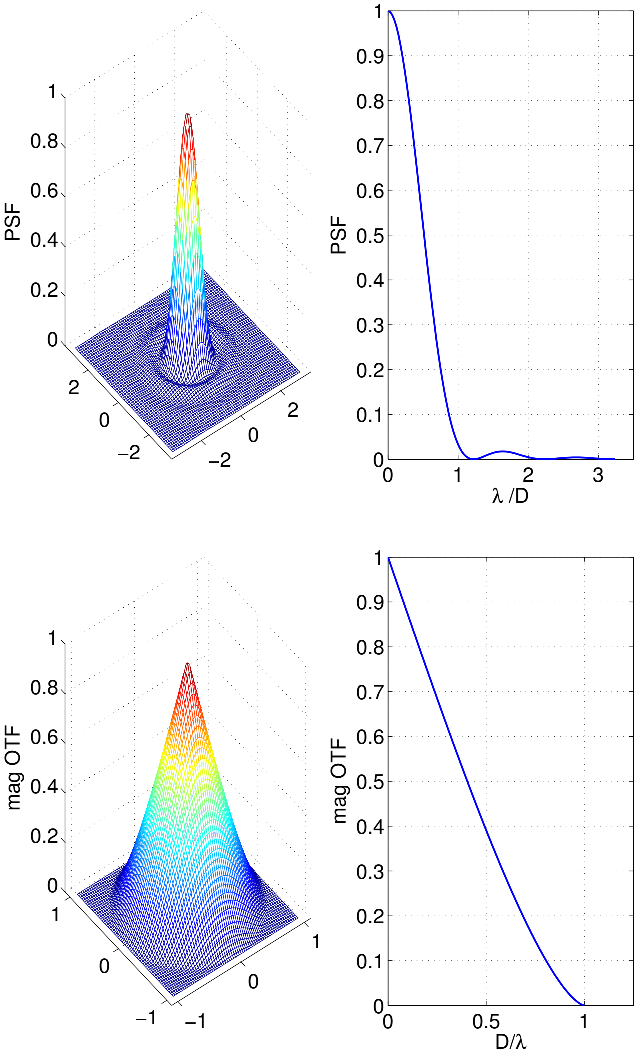


Figure 1: The PSF (top) and MTF (bottom) from a diffraction-limited optical system with a circular aperture. The limiting aperture has diameter D, and the light has wavelength λ.

The previous example assumed zero aberrations, but in reality no lens system is perfect. The issue of aberrations can be presented and modeled as simple phase deviations in the wavefronts of the light using MATLAB scripts that apply Zernike polynomials to the aperture function phase term. A comparison of the PSF and MTF, before and after a combination of two types of third-order monochromatic aberrations are simulated with MATLAB, is shown in Fig. 2. In this figure, the combined effect of both coma in x and astigmatism in y is modeled for the optical system, where each type of aberration is contributing a wavefront deviation of 2/10 of a wavelength. While combinations of multiple aberrations such as this can be complicated to visualize, it is easily investigated using MATLAB to apply a phase term to the generalized aperture function as dictated by the appropriate Zernike polynomials.

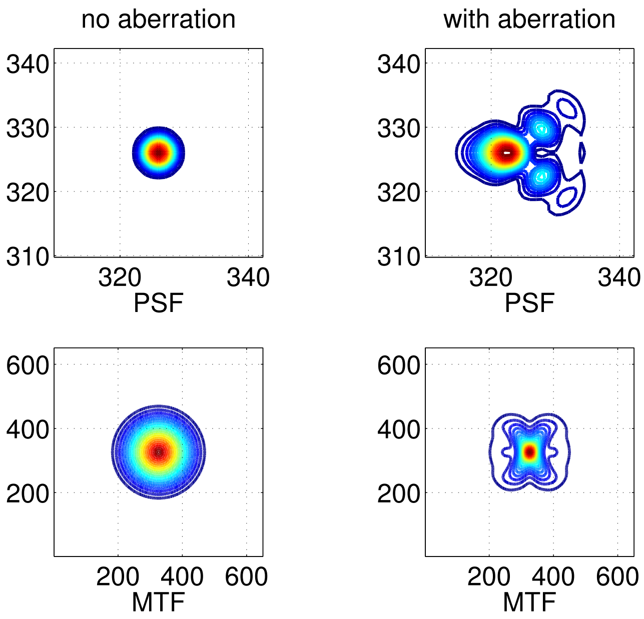


Figure 2: Comparison contour plots of the PSFs and MTFs without and with aberrations. The aberrations shown here represent a combination of coma in x and astigmatism in y. Note that to show sufficient detail, the PSF plots (top row) are "zoomed in" compared to the scale shown for the MTF plots (bottom row). Unlike Fig. 1, the units for the coordinate axes shown here are arbitrary, resulting from the padded size of the FFTs used to produce the plots.

The aperture function that combines the two types of aberrations shown in Fig. 2 is:

$$A(x_a, y_a) = A_0(x_a, y_a) e^{j2\pi\left(z_c\left[\sqrt{8}\left(3r^3-2r\right)\cos\theta\right]+z_a\left[\sqrt{6}\,r^2\sin(2\theta)\right]\right)} \quad (1)$$

where x_a and y_a represent the Cartesian coordinates at the aperture plane of the optical system, $r = \sqrt{x_a^2 + y_a^2}$, $\theta = \arctan(y_a/x_a)$, z_c is a constant that represents the "amount" of coma, and z_a is a constant that represents the "amount" of astigmatism, with both z_c and z_a expressed as a fraction of a wavelength. Zernike polynomials that can model

Table 1: The types of monochromatic aberrations associated with third-order optics. Note: the orthonormal representation of the Zernike polynomials is given.

Name	**Direction**	**Zernike polymomial**
distortion	x−tilt	$2r\cos\theta$
distortion	y−tilt	$2r\sin\theta$
field curvature or defocus	NA	$\sqrt{3}\left(2r^2-1\right)$
astigmatism	x	$\sqrt{6}\,r^2\cos(2\theta)$
astigmatism	y	$\sqrt{6}\,r^2\sin(2\theta)$
coma	x	$\sqrt{8}\left(3r^3-2r\right)\cos\theta$
coma	y	$\sqrt{8}\left(3r^3-2r\right)\sin\theta$
spherical	NA	$\sqrt{5}\left(6r^4-6r^2+1\right)$

third-order aberrations are listed in Table 1; similar polynomials can model fifth-order and seventh-order aberrations.

In each case, the students only need to add a straightforward phase term to the ideal aperture function A_0 where A is defined in the MATLAB script. The magnitude squared of the Fourier transform of the aperture function is the PSF, and the magnitude of the Fourier transform of the PSF is the MTF. . . and once again the existing knowledge of linear systems theory can be used to perform some very insightful optical engineering calculations.

Going beyond lens systems, the sensor array of a camera can be analyzed in a similar fashion, once again leveraging the students' existing knowledge of linear system theory, as shown Fig. 3. In this figure, the top plot is the MTF due to the finite size of an individual pixel. The next plot results from a common method of modeling the non-LSI (linear space invariant) effects of the spatial sampling of the sensor array. The next plot models the MTF due to an optical low-pass filter used by many camera manufacturers to control aliasing in the image. The bottom plot is the overall MTF of the sensor array.

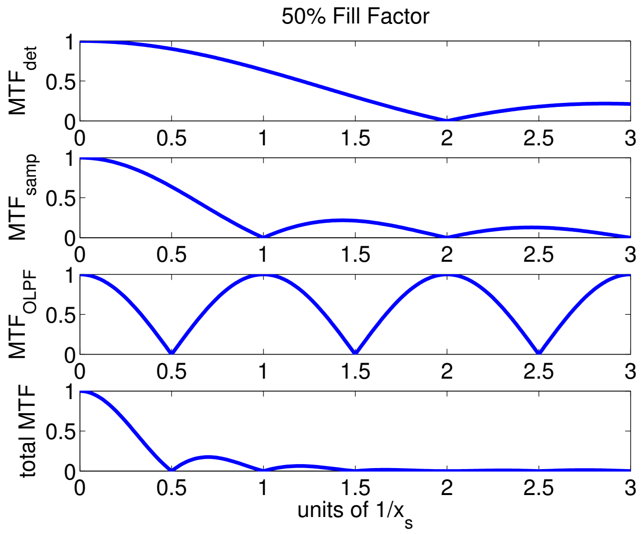
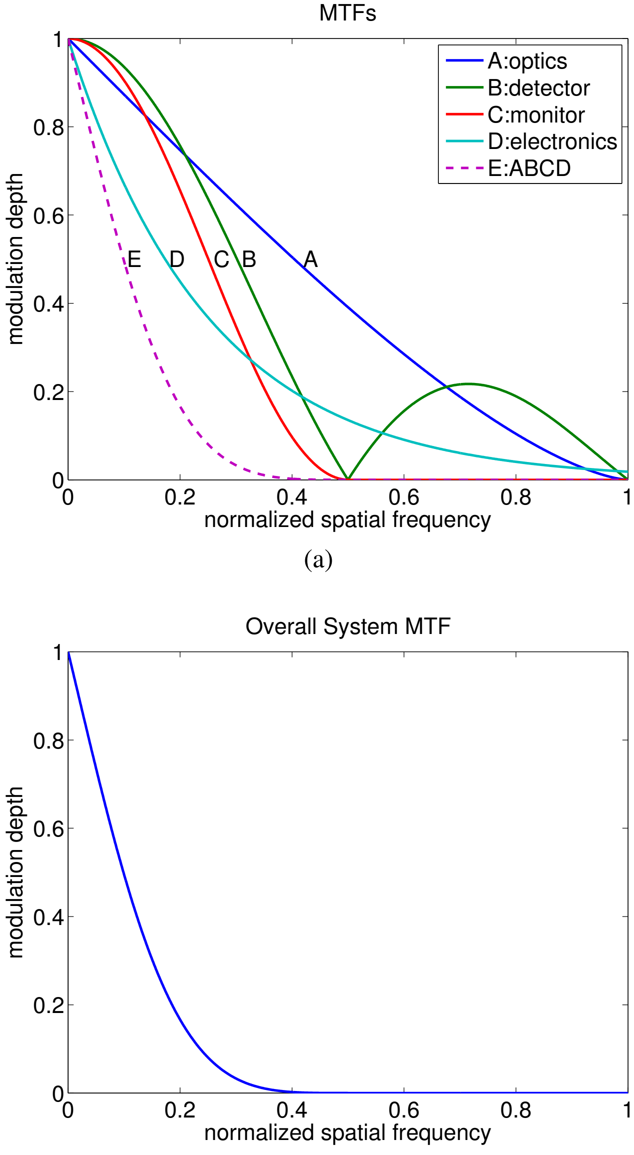


Figure 3: Individual MTFs along one dimension of a 50% fill factor sampled sensor array in a digital camera, with the product of all three (i.e., the overall total MTF) shown at the bottom. This example is for a common configuration of optical low-pass filter (OLPF). The horizontal axes are spatial frequency, labeled in $1/x_s$ units, where x_s is the size along that dimension of a single pixel in the sensor. The Nyquist frequency is thus labeled as 0.5.

Given MTFs of each part of a camera system, students can then combine all the parts to investigate the overall camera limitations. Suppose we have an imaging system that includes optics, a CCD detector array, and various electronics. We also want to display these images on a monitor screen. Each one of these four subsystems will have its own independent MTF. Given some reasonable assumptions, the overall system MTF will be the product of all four of the individual MTFs. This is shown in Fig. 4(a). The system MTF is shown alone in Fig. 4(b), to emphasize how different the system response can be, even if the optical cutoff frequency due only to the lens system is fairly high. While the lens system may be able to image certain fine lines and sharp edges in the object scene, much of this high frequency detail will never be recorded or show up on the monitor screen, due to the worse frequency response of the other MTFs. A realization of such limitations usually constitutes an epiphany for the students.



(a)

(b)

Figure 4: In (a), the independent MTFs for subsystems A through D are multiplied to obtain the overall system MTF shown as E. In (b), the overall system MTF is shown alone. In both subfigures, the spatial frequency axis is normalized with reference to the optical cutoff frequency D/λ.

In addition to the Fourier optics and MTF theory approach that takes full advantage of the students' prior knowledge of linear systems theory, the course also includes an algebraic treatment of concepts such as aperture, sensor, and pixel size, depth of field, field of view, reflection, refraction, and so forth. Given this, the students obtain a very practical working knowledge of optical engineering (via a single course) that is sufficient to support their research efforts.

Student Feedback and Results

The graduate course that we created (EE 5885 *Digital Image Formation and Acquisition*) using these methods has been taught twice so far (Spring 2013, Spring 2014) to University of Wyoming students from electrical, computer, mechanical, civil, and chemical engineering. Some survey items were presented to the students at the end of the semester in an attempt to assess whether our observations regarding the efficacy of the constructionism approach for a single course in optical engineering would be confirmed.

A 5-point Likert psychometric scale was used: 1–strongly disagree, 2–disagree, 3–neutral, 4–agree, and 5–strongly agree. Twelve students responded, and the mean results, rounded to two significant digits, are shown below for the items pertinent to this paper.

- I had very little or no background in optics or optical engineering prior to enrolling in this course. [4.5]
- This course took good advantage of my prior knowledge of topics such as linear systems theory and Fourier transforms. [3.9]
- I am now confident in my ability to quantitatively evaluate a digital camera system or similar type of imaging system. [4.2]

As can be seen, the students confirmed that the approach to the course described in this paper was effective in imparting to them a new understanding of optical engineering, and efficient in that it was accomplished with only a single course. More traditional approaches to teaching this material typically require at least two courses. Anecdotal feedback from a number of the graduate students who took this course reveals that they were able to immediately and effectively apply the key concepts

Table 2: Final exam scores from EE 5885, *Digital Image Formation and Acquisition*. Numbers shown are percent.

Final Exam	mean	std dev	median
Spring 2014	92.0	10.1	96.5
Spring 2013	92.0	4.7	93.0
Overall	92.0	6.5	94.5

of optical engineering as applied to their individual research areas. They were also able to confidently include both the associated theory and the specific optical engineering analysis of their individual test setups in their dissertations.

Another way to measure the effectiveness of this approach, as requested by peer reviewers of the draft manuscript of this paper, is by test scores. Results of the comprehensive final exam are shown in Table 2. Assuming the final exam is a valid measure of the students' mastery of the important concepts in the course, the use of the constructionism approach appears to be effective. From a comparison of the median and mean, the skew of the distribution is seen to be slightly toward the high end of the performance scale. However, the authors caution that any meaningful statistical inferences are unreliable due to the small sample size (n = 12).

Conclusions

The ease of facilitating student learning of a new topic area depends to a large extent on taking the best advantage their prior knowledge. By leveraging their existing cognitive frameworks, the students were less intimidated by the new subject, and more quickly established an acceptable level of expertise, as compared to "starting from scratch" in a new topic area.

While this may seem self-evident, confirmation of the technique is welcome. The challenge for faculty members who wish to take advantage of this method is to identify and make use of the most appropriate parts of the students' exiting knowledge base, and consciously structure a course from that perspective.

We observed a side benefit of this technique: many students developed a better appreciation for the interdisciplinary nature of engineering knowledge and expertise. The interactions among the students from different engineering majors was also of benefit to them.

While there are excellent books available that cover various parts of what was covered in this course, no existing book was found that spanned the entire camera or imaging system from end-to-end at an appropriate level of detail and rigor. At one end of the scale, there are some books aimed at camera and/or lens designers, and at the other end of the scale there are books aimed at photographers. Our target audience is made up of *technical users* of cameras and optics, who use such devices as a critical part of their work or research. To fill this need, a textbook was written expressly to support this course, and with the benefit of many helpful student suggestions, will be published soon.

References

1. H. Gardner, *The Development and Education of the Mind: The Selected Works of Howard Gardner*. World Library of Educationalists, New York: Routledge, 2006.
2. J. Piaget, *The Psychology of Intelligence*. New York: Routledge, 1950.
3. I. Harel and S. Papert, *Constructionism*. New York: Ablex Publishing, 1991.
4. M. Cakir, "Constructivist approaches to learning in science and their implications for science pedagogy: A literature review," *International Journal of Environmental & Science Education*, vol. 3, no. 4, pp. 193–206, 2008.
5. D. P. Ausubel, J. D. Novak, and H. Hanesian, *Educational Psychology: A Cognitive View*. Geneva, IL (USA): Holt McDougal, 2nd ed., 1978.

6. C. H. G. Wright and T. B. Welch, "Teaching real-world DSP using Matlab," *ASEE Comput. Educ. J.*, pp. 1–5, January–March 1999.

7. T. B. Welch, M. G. Morrow, and C. H. G. Wright, "Teaching practical hands-on DSP with MATLAB and the C31 DSK," in *Proceedings of the 2000 ASEE Annual Conference*, June 2000. Paper 1320-03.

8. T. B. Welch, C. H. G. Wright, and M. G. Morrow, "Poles and zeroes and Matlab, oh my!," *ASEE Comput. Educ. J.*, pp. 70–72, April–June 2000.

9. C. H. G. Wright, T. B. Welch, D. M. Etter, and M. G. Morrow, "Teaching DSP: Bridging the gap from theory to real-time hardware," *ASEE Comput. Educ. J.*, pp. 14–26, July–September 2003.

10. C. H. G. Wright, M. G. Morrow, M. C. Allie, and T. B. Welch, "Using real-time DSP to enhance student retention and engineering outreach efforts," *ASEE Comput. Educ. J.*, pp. 64–73, October–December 2008.

11. T. B. Welch, C. H. G. Wright, and M. G. Morrow, "The DSP of money," in *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, pp. 2309–2312, Apr. 2009.

12. T. B. Welch, C. H. G. Wright, and M. G. Morrow, *Real-Time Digital Signal Processing: From MATLAB to C with C6x DSPs*. Boca Raton, FL (USA): CRC Press, 2nd ed., 2012.

13. J. W. Goodman, *Introduction to Fourier Optics*. New York: McGraw-Hill, 2nd ed., 1996.

14. G. D. Boreman, *Modulation Transfer Function in Optical and Electro-Optical Systems*, vol. TT52 of *Tutorial Texts in Optical Engineering*. Bellingham, WA (USA): SPIE Press, 2001.

15. G. C. Holst and T. S. Lomheim, *CMOS/CCD Sensors and Camera Systems*. Bellingham, WA (USA): SPIE Press, 2nd ed., 2011.

16. R. H. Vollmerhausen, D. A. Reago, Jr., and R. G. Driggers, *Analysis and Evaluation of Sampled Imaging Systems*. Bellingham, WA (USA): SPIE Press, 2010.

Biographical Information

Cameron H. G. Wright, Ph.D, P.E., is with the Department of Electrical and Computer Engineering at the University of Wyoming, Laramie, WY. His research interests include signal and image processing, real-time embedded computer systems, biomedical instrumentation, and engineering education. He is a member of ASEE, IEEE, SPIE, BMES, NSPE, Tau Beta Pi, and Eta Kappa Nu. E-mail: `c.h.g.wright@ieee.org`

Thad B. Welch, Ph.D, P.E., is with the Department of Electrical and Computer Engineering at Boise State University, ID. His research interests include implementation of communication systems using DSP techniques, DSP education, multicarrier communication systems analysis, and RF signal propagation. He is a member of ASEE, IEEE, Tau Beta Pi, and Eta Kappa Nu. E-mail: `t.b.welch@ieee.org`

Michael G. Morrow, P.E., is with the Department of Electrical and Computer Engineering at the University of Wisconsin, Madison, WI. His research interests include real-time digital systems, embedded system design, software engineering, curriculum design, and educational assessment techniques. He is a member of ASEE and IEEE. E-mail: `morrow@ieee.org`