PRELIMINARY ANALYSIS OF COGENERATION HEAT EXCHANGER CONFIGURATIONS

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Abstract

A matrix method (normally involving computer solution) is presented for calculating the overall effectiveness of multiple heat exchangers employed in a microturbine cogeneration application, but is generally applicable to any choice or combination of prime movers; the heat exchanger configurations considered include series and parallel flow arrangements. The simultaneous satisfaction of individual heat exchanger effectiveness and energy conservation are used to determine the minimum number of heat exchangers required to produce hot water at a specified temperature and flowrate. The overall effectiveness of the different arrangements is defined in terms of the percentage capture of the microturbine exhaust gas energy; this effectiveness depends on the characteristics of the microturbine, the working fluids, and the required water flow rate and delivery temperature. Detailed results are provided for two of the candidate heat exchanger configurations, with an explanation of the general procedure for analysis of other arrangements. In addition to overall effectiveness and number of required units, entropy production is also utilized as a basis of comparison. Given the assumptions presented herein, the analysis procedure allows the preliminary comparison of alternative cogeneration heat exchanger configurations. With the number of heat exchangers required to satisfy heating demand, a preliminary economic impact of each configuration may then be made. Recommendations are made for possible incorporation into an undergraduate curriculum.

Nomenclature

A	=	$\varepsilon C_{min}/C_{cold}$
B	=	C_{min}/C_{cold}
c	=	specific heat (Btu/lbm·ºR, kJ/kg·K)
C	=	$\dot{m}c$, capacity rate (Btu/s·ºR, kW/K)
C'_{min}	=	minimum capacity rate for the entire assembly of heat exchangers
EGU	=	N_s, number of entropy generation units
$\dot{m}$	=	mass flow rate (lbm/s, kg/s)
N_s	=	$\dot{S}/C'_{min}$, number of entropy generation units
n_{GTE}	=	number of gas turbine engines (prime movers)
p	=	pressure (lbf/in^2, Pa)
R_h	=	gas constant for air (Btu/lbm·ºR, kJ/kg·K)
S	=	entropy (Btu/ºR, kW-s/K)
$\dot{S}$	=	entropy rate (Btu/s·ºR, kW/K)
T	=	temperature (ºF, ºR, ºC, K)

Greek

α = $\Delta p / p_i$, ratio of heat exchanger pressure drop to inlet pressure

ε = $\dfrac{C_c\left(T_{co} - T_{ci}\right)}{C_{min}\left(T_{hi} - T_{ci}\right)}$, heat exchanger effectiveness

Δ = change in quantity

Subscripts

atm = ambient conditions
avg = output is the mass average of inputs
c = cold fluid (water)
GTE = gas turbine engine
h = hot fluid (gas)
i = inflow
max = maximum
min = minimum
o = outflow
overall = including all heat exchangers in the assembly
p = constant pressure condition
required = design criterion for hot water temperature delivered
start = condition of water entering the entire heat exchanger assembly

Introduction

Cogeneration is becoming increasingly popular as a means of simultaneously satisfying not only electrical power demands, but also heat production requirements through the effective use of waste heat from a prime mover[1]. In particular, a distributed system employing cogeneration is being evaluated for possible implementation at the United States Air Force Academy, with the ultimate intent of replacing the existing hot water plant that supplies the entire Academy through an inefficient distribution system. As part of this analysis, hot water temperature and flow rate were specified at the inlet to each building. This translated into a required discharge temperature from the cogeneration heat exchangers, which, in turn, set the number and type of prime movers (microturbines in this analysis) and heat exchangers required. Depending on the water flow path through the heat exchangers, calculation of the number and type of prime movers proved to be more difficult than first thought. Once obtained, however, the remainder of the system performance, including life cycle analyses, could proceed.

ε=0.88		
	Hot (Gas)	**Cold (Water)**
$\dot{m}\left(\frac{lbm}{s} \Big/ \frac{kg}{s}\right)$	0.80/0.36	8.00/3.63
$c\left(\frac{Btu}{lbm\cdot{}^\circ R} \Big/ \frac{kJ}{kg\cdot K}\right)$	0.24/1.00	1.00 /4.19
$C\left(\frac{Btu}{s\cdot{}^\circ R} \Big/ \frac{kJ}{s\cdot K}\right)$	0.19/0.37	8.00/15.2
$T_{start}\left({}^\circ F/{}^\circ C\right)$		170/77
$T_{required}\left({}^\circ F/{}^\circ C\right)$		300/ 149
$T_{GTE}\left({}^\circ F/{}^\circ C\right)$	550/288	
Single Heat Exchanger Pressure Drop, $\alpha = \Delta p/p_{hi}$	0.05	

Table 1: Given system parameters.

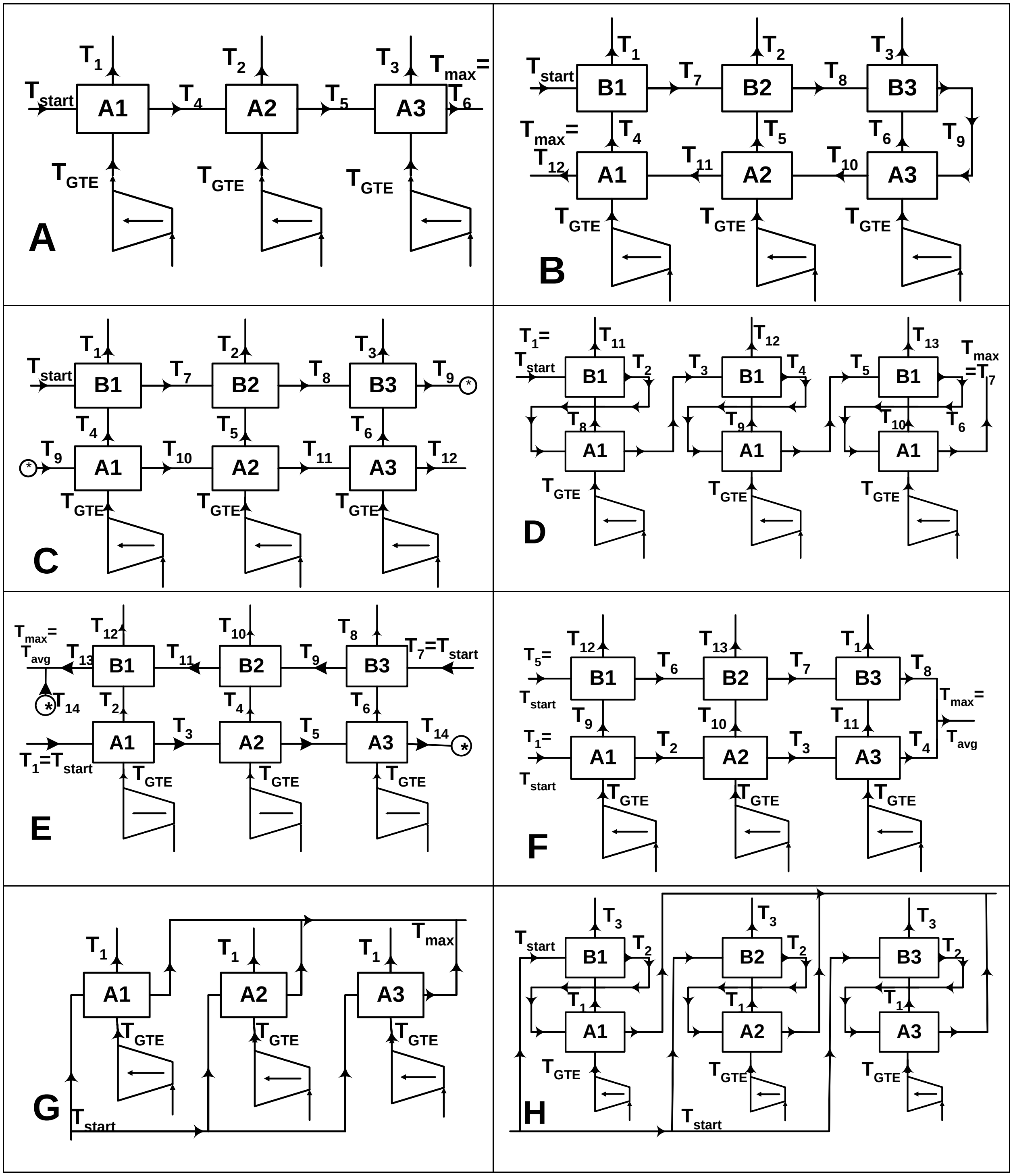


Table 2: Configuration comparison for $n_{GTE} = 3$ (Note: For configurations G and H, water flow divided into equal flows to each prime mover).

A method of performing preliminary analyses of different cogeneration heat exchanger configurations is presented. It assumes that the heat exchanger effectiveness and exhaust gas mass flow rate and temperature for each prime mover are known from manufacturer-specified data; the required water delivery temperature and mass flow rate are based upon the heating system demand. With this information in hand (see Table 1), different flow configurations may be considered, including both series and parallel heat exchanger arrangements. The results of this analysis allow the field of competing system configurations to be narrowed for further, more detailed analysis.

Assumptions

Basic assumptions used in this simple comparative analysis include the following:

- Steady state operation
- Water is incompressible
- A pressure drop of $\alpha = \Delta p/p_i = 0.05$ is assumed on each heat exchanger's gas side (p_i is the hot gas pressure at the heat exchanger inlet).
- No stray heat transfer from each prime mover's heat exchanger(s) or from interconnecting piping
- Prime mover exhaust is modeled as an ideal gas (air)
- Effects of back pressure on prime mover performance are neglected, as are compressibility effects
- Constant specific heat for each fluid in the analysis
- All prime movers (microturbines, in the current analysis) are identical, and produce the same exhaust temperature
- Heat exchanger effectiveness, ε, is identical for all individual units.
- Each prime mover can be outfitted with one or two heat exchangers

Configurations Considered

The various configurations that were evaluated are shown in Table 2 (see previous page). These ranged from a single-pass, straight-through path, to more complicated configurations. For the simpler configurations, the water discharge temperature (T_{max}) for a particular number of heat exchangers was readily calculated; since the inlet water temperature and heat exchanger effectiveness were both known for each heat exchanger, calculations were able to be conducted in a sequence which mirrored the water flow path. For the more complicated flow paths, however, the interrelationships of temperatures required simultaneous solution of the pertinent equations via computer matrix inversion.

Cold path (water) flow configurations which were investigated included (letters correspond to those in Table 2):

A. Single path: straight through (only configuration with a single heat exchanger on each prime mover)
B. Single path: looped back on itself in "counter flow" configuration
C. Single path: after flow exits from the "B" bank of heat exchangers, it returns to the first prime mover for passage through the "A" bank, resulting in a "parallel flow" configuration
D. Single "serpentine" path: through each pair of heat exchangers on each prime mover (passes through the "B" heat exchanger, then through the "A" heat exchanger)—flow then moves to the next prime mover
E. Two paths: flow divided equally in "counter flow" configuration; fluid streams mixed at output
F. Two paths: flow divided equally in "parallel flow" configuration; fluid streams mixed at output
G. Flow divided equally among all prime movers and then reformed; single heat exchanger on each prime mover
H. Flow divided equally among all prime movers and then reformed; two heat exchangers on each prime mover

Where the cold stream is split (as in configurations E and F), it does not necessarily have to be equally divided. Additional flow configurations are possible, limited only by imagination. The same analysis technique described herein may be employed to evaluate each configuration to ascertain the minimum number of heat exchangers/prime movers necessary to achieve the desired outlet temperature.

Analysis

For each heat exchanger, the solution must satisfy the equations for heat exchanger effectiveness [1]

$$\varepsilon = \frac{C_c\left(T_{co} - T_{ci}\right)}{C_{min}\left(T_{hi} - T_{ci}\right)} \tag{1}$$

and for energy conservation

$$C_c\left(T_{co} - T_{ci}\right) = C_h\left(T_{hi} - T_{ho}\right). \tag{2}$$

Here, the subscripts stand for cold water (c), hot gas (h), inlet (i), and outlet (o). Additionally, the capacity rate for a particular fluid is the product of its mass flow rate $\dot{m}$, and its specific heat, c:

$$C = \dot{m}c. \tag{3}$$

C_{min} is the minimum of C_h and C_c.

For the following configurations, the calculation of the cold stream discharge temperature is straightforward and sequential:

A. T_{co} and T_{ho} are solved using Eqs. (1) and (2) moving from one unit to the next.

D. Although the water path may appear somewhat complicated, the configuration for one prime mover and its associated heat exchangers may be analyzed using Eqs. (1) and (2) to calculate T_{co} and T_{ho}; Eq. (1) may then be used to calculate an equivalent heat exchanger effectiveness for that unit, and the problem then reduces to that of configuration A.

E. Analysis of this dual-path configuration proceeds in a very straightforward fashion by considering the passage of half of the cold fluid through the "A" bank of heat exchangers first, and then following the other half through the "B" bank of heat exchangers, each time using Eqs.(1)and (2) The two fluid streams are then mixed at the outlet, and the temperature is calculated as a mass-weighted average.

F. Calculations for this configuration are very similar to configuration E.

G. The solution is very straightforward, as was done for configuration A.

While these four configurations are rather simple to analyze, they might not prove to yield the most efficient option; rather, an arrangement where the cold fluid is heated more slowly (minimizing temperature differences between hot and cold fluid streams at any point to minimize entropy production based on second law considerations) is more likely to be employed. This is precisely the scheme employed in configurations B and C; each will require simultaneous solution of the relevant equations. The setup for solution will be displayed for the "parallel flow" configuration (Table 2C); the "counter flow" configuration (Table 2B) is evaluated in a similar fashion.

Consider n_{GTE} = 3 prime movers, each outfitted with two heat exchangers (Table 2C); the technique may be applied to any number of prime movers, with the number increased successively until the desired output temperature is obtained. The hot (gas) exhaust temperature from each prime mover is T_{GTE}, and the cold (water) temperature into the first heat exchanger from the building heating loop is T_{start}.

To illustrate calculations using these configurations and this method, values are used from Table 1 for C_c (water), C_h (gas), and an individual heat exchanger effectiveness, ε, for a representative building heating application whose required heating system temperature is $T_{required}$ = 300ºF (149ºC). It should be noted that a judicious choice of subscripts when numbering temperatures will greatly simplify the generation of equations in the solution which follows; this choice usually results in banded sub-matrices and a symmetry which facilitates matrix generation within a computer program.

By rearranging Eqs. (1) and (2), expressions for the outlet temperature for each fluid at each heat exchanger may be obtained:

$$T_{co} = T_{ci} + \varepsilon \frac{C_{min}}{C_c}\left(T_{hi} - T_{ci}\right) \qquad (4)$$

$$T_{ho} = T_{hi} - \frac{C_c}{C_h}\left(T_{co} - T_{ci}\right) \qquad (5)$$

Letting $A = \varepsilon C_{min} / C_{cold}$ and $B = C_{cold} / C_{hot}$, and then applying these last two equations to each of the six heat exchangers, the following system of simultaneous equations in matrix format is obtained:

$$\begin{pmatrix} 0 & 0 & 0 & -A & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -A & 0 & A-1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -A & 0 & A-1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A-1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A-1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A-1 & 1 \\ 1 & 0 & 0 & -1 & 0 & 0 & B & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & -B & B & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & -B & B & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -B & B & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -B & B & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -B & B \end{pmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \end{Bmatrix} = \begin{Bmatrix} (1-A)\,T_{start} \\ 0 \\ 0 \\ A\,T_{GTE} \\ A\,T_{GTE} \\ A\,T_{GTE} \\ B\,T_{start} \\ 0 \\ 0 \\ A\,T_{GTE} \\ A\,T_{GTE} \\ A\,T_{GTE} \end{Bmatrix} \qquad (6)$$

A configuration of n_{GTE} prime movers results in a matrix of order $4n_{GTE}$. Solving this system of equations (usually via computer, due to the size of matrices which may be produced) results in a maximum hot water temperature of $T_{max} = T_{12} = 218$ºF (103ºC); adding successively more units to this system ultimately results in $n_{GTE} = 16$ units (for a total of 4·16 = 64 subscripted temperatures) meeting the design criterion of $T_{required} = 300$ºF (149ºC) for this example with $T_{max} = T_{64} = 305$ºF (152ºC).

It is useful to calculate the number of dimensionless entropy units generated, N_s, defined as the entropy, S, produced per unit mass of fluid flowing, that fluid being the one which possesses the minimum capacity rate for the entire bank of heat exchangers. If a control volume is considered around the entire heat exchanger assembly, an adiabatic, steady state process for $n_{GTE} = 3$ prime movers and $T_{max} = T_{12}$ results in an entropy production rate of $\dot{S}$[2,3], and a nondimensional number of entropy generation units of (see Refs.[5,6])

$$N_s = \frac{\dot{S}}{C'_{min}} = \frac{\dot{m}_c c_c}{C'_{min}} \ln\left(\frac{T_{max}}{T_{start}}\right) + \frac{\dot{m}_h}{C'_{min}}\left\{c_h\left[\ln\left(\frac{T_1}{T_{GTE}}\right) + \ln\left(\frac{T_2}{T_{GTE}}\right) + \ln\left(\frac{T_3}{T_{GTE}}\right)\right] - R_h \ln\left(\frac{p_{atm}}{p_{GTE}}\right)\right\} \qquad (7)$$

All temperatures in this last equation must be absolute. Additionally, the overall hot fluid (gas) capacity rate is now calculated as $n_{GTE}\dot{m}_h c_h$; C'_{min} is calculated as the minimum of the cold fluid flow and the overall hot fluid flow capacity rates. Note that, for a single bank of heat exchangers (as in configuration A), $p_{atm}/p_{GTE} = 1-\alpha$, where α is the percentage pressure drop on the hot gas side for a single heat exchanger; for all other configurations considered herein, each prime mover is configured with two heat exchangers, so that $p_{atm}/p_{GTE} = (1-\alpha)^2$. Finally, an overall effectiveness for the entire bank of heat exchangers may be calculated as

$$\varepsilon_{overall} = \frac{C_c\left(T_{max} - T_{start}\right)}{C'_{min}\left(T_{GTE} - T_{start}\right)}. \qquad (8)$$

For configuration C, 16 prime movers are required to satisfy the minimum hot water temperature criterion, with an associated nondimensional entropy production of N_s = 0.107 and $\varepsilon_{overall}$ = 0.927.

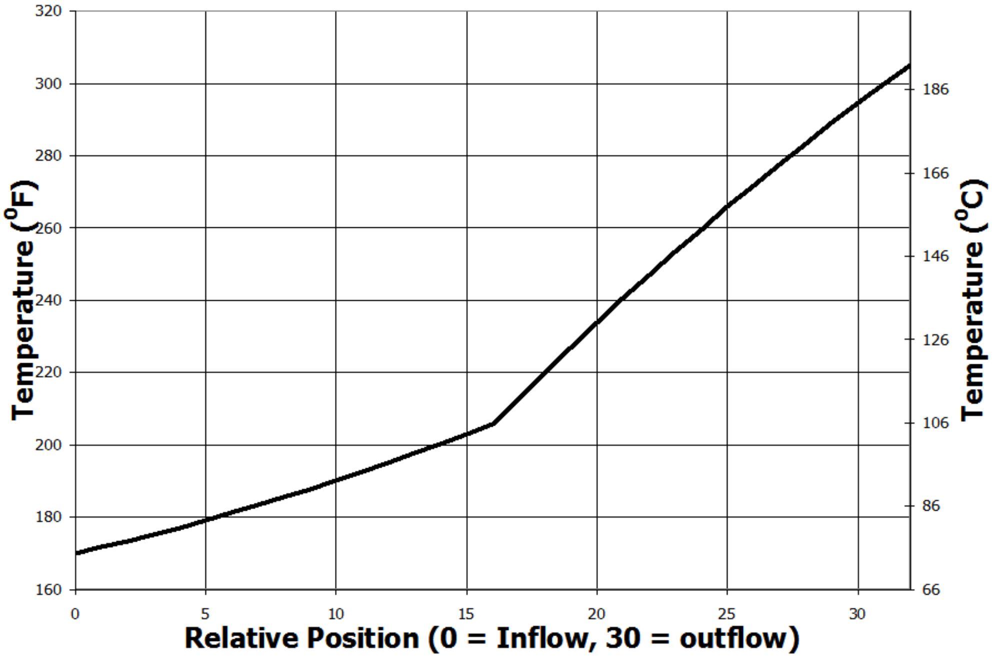


Figure 1: Water delivery temperature as a function of distance along flow path, "parallel" flow configuration.

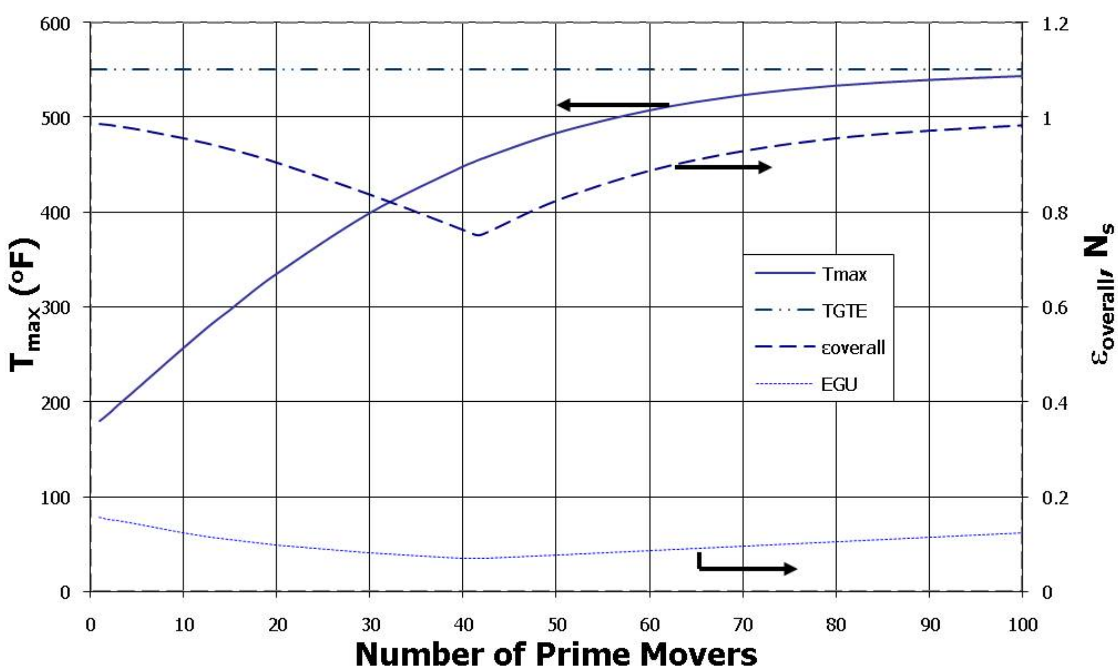


Figure 2: Water delivery temperature as a function of number of prime movers, "parallel" flow configuration.

The increase in cold fluid temperature is shown in Fig. 1 as a function of relative position through the heat exchangers; the "knee" in the curve is associated with the water flow transitioning from the "B" bank of heat exchangers to the "A" bank. Fig. 2 depicts the increase in water outlet temperature as a function of the number of prime movers; with an increasing number of heat prime movers, the curve asymptotically approaches T_{GTE} = 550ºF (288ºC). Also shown in Fig. 2 as functions of the number of prime movers are N_s, and $\varepsilon_{overall}$; each of these curves displays a minimum value at the point where C'_{min} changes from $n_{GTE}\dot{m}_h c_h$ to $\dot{m}_c c_c$, between n_{GTE} =41 and 42 for this configuration.

With the "counterflow" configuration of 3 prime movers depicted in Table 2B, T_{max} = 218ºF (103ºC) is achieved; again, it is found that 16 prime movers are required to satisfy the supply temperature criterion with T_{max} = 304ºF (151ºC). This temperature is less than that produced by the same number of primer movers used in configuration C; additionally, the nondimensional entropy production (N_s = 0.108) is greater than that obtained in configuration C. Not surprisingly, the overall effectiveness ($\varepsilon_{overall}$ = 0.918) is less than that achieved by configuration C.

Table 3 synopsizes the results from the analyses of the various configurations. Normally, the number of prime movers would be the principal evaluation criterion, but in the case of configurations where the minimum number of prime movers is equal (configurations B, C, and F), the outlet water temperature (T_{max}), entropy production, and overall effectiveness may be used as additional criteria.

In this particular problem, configuration H was chosen. Not only does this heat exchanger configuration provide the highest effectiveness, the lowest production of specific entropy, and require the fewest units of those configurations considered, it also allows the same flow path configuration on each unit.

Once a choice is made, further analyses would be required for a more detailed design. It would be prudent to consider various models of heat exchanger (each, possibly, with a different effectiveness), to vary pinch temperatures, and to ensure that acid dew points are not attained in the cooler sections of each heat exchanger. Perhaps recirculation of some of the heated water to the inlet of each heat exchanger might even be considered in the effort to minimize the potential for acid formation. Again, these and many other issues would have to be considered in a more detailed design which would follow.

The same methodology could be employed with combinations which include different prime movers, different heat exchangers, and (for multiple flow paths) unequal division of flow. The matrices associated with such configurations, however, would not necessarily be as easy to generate automatically within a program.

Conclusion

A matrix inversion (suitable for computer solution) technique to evaluate alternative configurations of heat exchangers in cogeneration plants is presented, and is based on a required flow rate and delivery temperature for the cold fluid; hot fluid criteria (prime mover flow rate and exhaust gas temperature) and individual heat exchanger effectiveness are obtained from manufacturer-specified data for the prime mover. The basic definition of heat exchanger effectiveness and the requirement for energy conservation on each heat exchanger are combined to calculate temperatures at all points in the system, most importantly at the cold fluid exit from the last heat exchanger. The number of prime movers is changed until the heating system demand temperature is achieved or exceeded. Should alternative configurations result in equal numbers of prime movers satisfying the demand temperature criterion, additional criteria are offered to better identify the best configuration. Within the limits of the assumptions, this methodology provides an

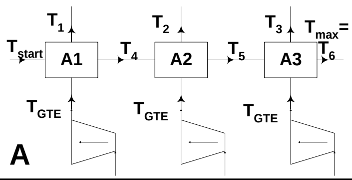
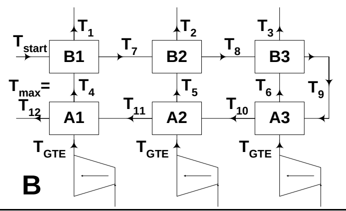
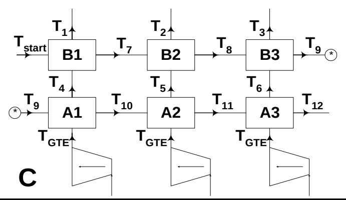
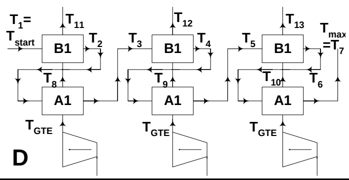
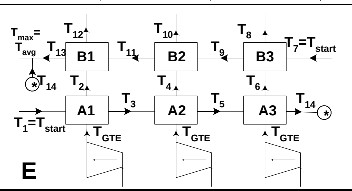
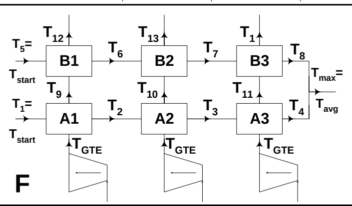
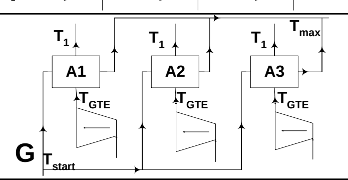
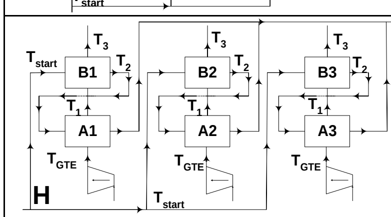
Configuration	n_{GTE}	T_{max} (°F/°C)	$\varepsilon_{overall}$	N_s
	20	302/150	0.724	0.092
	16	304/151	0.918	0.108
	16	305/152	0.927	0.107
	18	303/151	0.810	0.108
	17	302/150	0.850	0.109
	16	300/149	0.893	0.109
	17	306/152	0.880	0.116
	15	304/151	0.980	0.105

Table 3: Analysis results.

effective method for making preliminary comparisons of alternative configurations. The results of this thermodynamic analysis may then be used to calculate the economic impact of choosing a particular configuration, including the capital costs associated with the number of prime movers and heat exchangers.

This analysis was originally conducted with a cadet engaged in an independent study performed in conjunction with a feasibility study on the implementation of cogeneration at the United States Air Force Academy. Students typically find the concept of entropy generation to be esoteric and without any useful application—this method of analysis demonstrates an application of theory to a meaningful problem. It would be an ideal case study for students in a classical thermodynamics course (typically taken in the sophomore year at most undergraduate institutions, but in the junior year at the Academy as part of its thermo-fluids curriculum). Additionally, the method of analysis could be made part of a term project in a more extensive higher-level (typically senior or fifth-year) course in energy conversion or green energy.

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Biographical Information

Michael Maixner graduated with distinction from the U. S. Naval Academy, and served as an officer in the USN for 25 years; his first 12 years were spent as a shipboard officer, while his remaining service was spent strictly in engineering assignments. He received his Ocean Engineer and SMME degrees from MIT, and his Ph.D. in mechanical engineering from the Naval Postgraduate School. He served as an Instructor at the Naval Postgraduate School and as a Professor of Engineering at Maine Maritime Academy; he is currently a member of the Department of Engineering Mechanics at the U.S. Air Force Academy.

Colton Heaps, a distinguished graduate from the Air Force Academy, completed his undergraduate degree in mechanical engineering in 2005. He was one of four Public Service Fellows to attend the Harvard Kennedy School where he earned a Master of Public Policy in 2007. He is a civil engineering officer and currently stationed at Shaw Air Force Base where he is serving as the Executive Officer to the Director of Installations for US Air Force Central Command. He has earned the professional certifications of Project Management Professional and Leadership in Energy and Environmental Design Accredited Professional.