

A GRAPHICAL USER INTERFACE (GUI) FOR A UNIFIED APPROACH FOR CONTINUOUS-TIME COMPENSATOR DESIGN

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Abstract

As an alternative to the numerous distinct controller design algorithms in classical control textbooks, a simple unified design approach, which is independent of the form of the linear system information, was developed in previous work for all standard classical compensators. This approach is based on a simple root locus design procedure for a proportional-derivative (PD) compensator. From this procedure, design procedures for unified notation lead, proportional-integral (PI), proportional-integral-derivative (PID), and PI-lead compensators were developed. With this proposed approach, students can concentrate on the larger control system design issues, such as compensator selection and closed-loop performance evaluation, rather than the intricacies of a particular design procedure.

Once students learn the unified design process discussed above, it is important that they get an opportunity to apply it to design and laboratory projects. Most real life examples require design iterations. The Graphical User Interface (GUI) developed in this paper not only makes this feasible, but also makes this an excellent learning opportunity. The authors have implemented the unified compensator design procedure as a GUI in MATLAB. The GUI presents the user with both root locus and Bode information. Either domain can be used for design. The effect of the design on both domains can be seen instantaneously. The GUI also provides the user with the closed-loop step and Bode responses as well. Design specifications in the time and frequency domain are easily verified.

Introduction

In controls education today, there seems to be a gap between the theory taught in the typical undergraduate classroom and the skills required for practical application of control systems. One obvious reason for this is the lack of undergraduate control system laboratories. The control systems community has recognized this need.[1,2] In many departments around the world, undergraduate control system laboratories are being developed.[3,4] A less obvious reason for this gap is the “cookbook” approach to compensator design found in typical classical control textbooks.[5,6,7] For example, a quick comparison reveals significant differences in the procedures for root locus lead design and root locus Proportional-Integral (PI) design. Even more importantly, there are significant differences in the procedures for lead compensator design using root locus techniques and those using Bode techniques. Furthermore, for even simple systems, these design procedures may yield poor results.[8] Therefore, students end up concentrating on the different “recipes” that may or may not yield satisfactory results, and, consequently, tend to miss the “big picture”.

In previous work, design methods were developed that permit students to apply a simple, unified design approach for six standard compensators, Proportional-Derivative (PD), lead, Proportional-Integral (PI), lag, Proportional-Integral-Derivative (PID), and PI-lead independent of the form of the system information.[9,10] In root locus design, the computational procedures are based on the open-loop transfer function whereas, in Bode design, the computational procedures are based on the magnitude and phase of the open-loop frequency response. With this approach,

students can concentrate on the larger control system design issues, such as compensator selection and closed-loop performance evaluation, rather than the intricacies of a particular design procedure. The unified design approach in this paper is based on a simple procedure for root locus Proportional-Derivative (PD) design. Design procedures for the other five compensators are based on this simple PD design procedure. The unified design approach has been applied successfully in classical control classes at the U.S. Naval Academy.

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A classical control design GUI is included in the MATLAB Control System Toolbox. It is called the SISO Design Tool, where SISO refers to Single-Input Single-Output transfer functions.[11] Like the GUI in this paper, the SISO Design Tool allows the student to view both root locus and Bode information. Unlike the GUI in this paper, the built in design tools are not based on the unified design procedures of the author's previous work.[9,10] Consequently, it does not provide the same seamless integration for the students as they transition from simple classroom exercises to more complicated lab and design projects.

Compensator Design

The integrated design procedure using time or frequency domain plant data requires a generalization of the angle criterion from root locus design. A standard closed-loop system is shown in Figure 1 where K is the control gain, $G_c(s)$ is the compensator and $G_p(s)$ represents the plant dynamics.

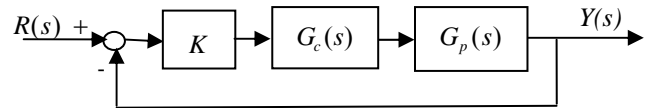


Fig. 1. Closed-loop block diagram.

In root locus design, the compensator must satisfy the well-known angle and magnitude criteria

$$(1) \quad \begin{aligned} \angle G_c(s_d) + \angle G_p(s_d) &= \pm 180^\circ \\ K |G_c(s_d)G_p(s_d)| &= 1 \end{aligned}$$

where the design point can be written as $s_d = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$ in terms of the desired damping ratio ζ and the desired natural frequency ω_n .

In Bode design methods, the specifications are incorporated through the desired phase margin PM and the gain crossover frequency ω_{gc} and result in another set of angle and magnitude constraints

$$(2) \quad \begin{aligned} \angle G_c(j\omega_{gc}) + \angle G_p(j\omega_{gc}) &= \pm 180^\circ + \text{PM} \\ K |G_c(j\omega_{gc})G_p(j\omega_{gc})| &= 1 \end{aligned}$$

Using standard 2nd order assumptions,[5] the PM and ω_{gc} can also be determined from the continuous-time design point as

$$(3) \quad PM = \tan^{-1} \left(\frac{2\zeta}{\sqrt{-2\zeta + \sqrt{1+4\zeta^4}}} \right)$$

$$(4) \quad \omega_{gc} = \frac{2\zeta\omega_n}{\tan(PM)}$$

Equations (1) and (2) can be combined to get the generalized angle and magnitude constraints

$$(5) \quad \begin{aligned} \angle G_c(s_0) + \angle G_p(s_0) &= \pm 180^\circ + \phi_0 \\ K |G_c(s_0)G_p(s_0)| &= 1 \end{aligned}$$

where the generalized design point is

$$(6) \quad s_0 = \begin{cases} s_d, & \text{root locus} \\ j\omega_{gc}, & \text{Bode} \end{cases}$$

and the desired angle in the angle constraint is

$$(7) \quad \phi_0 = \begin{cases} 0, & \text{root locus} \\ PM, & \text{Bode} \end{cases}$$

Using the angle constraint in (3), the desired compensator angle θ_c can be computed from the plant information and the design point without knowledge of the compensator type.

$$(8) \quad \theta_c := \angle G_c(s_0) = \pm 180^\circ + \phi_0 - \angle G_p(s_0)$$

In root locus methods, θ_c determines a geometric relationship between the design point and the compensator poles and zeros. In Bode methods, θ_c is the phase that must be added at the gain crossover frequency.

PD Compensator

In this unified method, the design procedures for all compensators are based on the PD design procedure.[9.10] The PD compensator has a transfer function $G_c(s) = s + z$ where the zero is chosen to meet the design specifications. The

angle of the PD compensator at the design point is

$$(9) \quad \theta_c = \angle G_c(s_0) = \angle(s_0 + z)$$

Therefore the compensator zero is given by

$$(10) \quad z = \sigma_0 + \frac{\omega_0}{\tan(\theta_c)}$$

where $s_0 = -\sigma_0 + j\omega_0$ is the design point in the unified notation. For this compensator, and each compensator to follow, the gain K is computed using the magnitude constraint in (3).

There is a limit to the improvement that the PD compensator can achieve. In general, the compensator zero should be minimum phase because a non-minimum phase compensator can lead to poor performance and/or instability in the closed-loop system. Under this assumption, the maximum value for θ_c is $\angle(s_0)$ and is achieved by the derivative compensator $G_c(s) = s$. As z approaches infinity, θ_c approaches 0. It follows that the design point can be achieved or, equivalently, that the PD compensator design problem is feasible if and only if $0 \leq \theta_c \leq \theta_{c, \max}$, where $\theta_{c, \max} = \angle s_0$. Feasibility relationships are shown in Table 1 for other compensators (lead, PI, PID, PI-lead) using relationships between these compensators and the PD compensator.

Table 1. Feasibility of Compensator Types	
Compensator	Feasibility relationship
PD, lead	$0 \leq \theta_c \leq \theta_{c, \max}$
PI, lag	$-\theta_{c, \max} \leq \theta_c \leq 0$
PID, PI-lead	$-\theta_{c, \max} \leq \theta_c \leq \theta_{c, \max}$

Lead Compensator

The lead (practical PD) compensator has a transfer function $G_c(s) = \frac{s+z}{s+p}$ where $z < p$.

The angle of lead compensator at the design point s_0 is

GUI Design

$\angle G_c(s_0) = \angle(s_0 + z) - \angle(s_0 + p) = \theta_z - \theta_p$ and the lead pole and zero must be selected to satisfy the angle constraint (7), or equivalently, $\theta_c = \theta_z - \theta_p$.

For a given desired compensator angle θ_c , the angle contributed by the lead compensator zero θ_z must be greater than that for a PD compensator due to the angle contributed by the lead compensator pole θ_p . Figure 2 provides a graphical comparison of the pole and zero locations of the PD compensator and those of the lead compensator. It follows that the PD compensator angle is obtained if $p \rightarrow \infty$. In this sense, the PD compensator design establishes limits on the lead compensator design. Specifically, the lead zero must be smaller than the PD zero, i.e., $0 \leq z_{lead} \leq z_{pd}$.

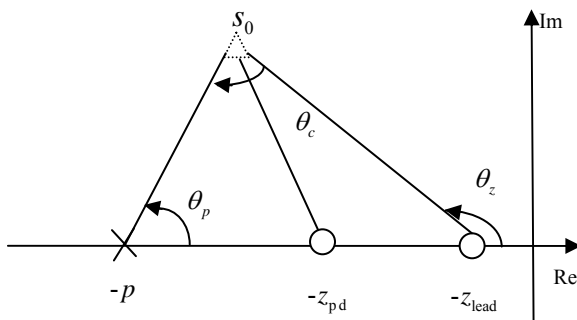


Fig. 2. Relationship between PD and lead compensators.

After the lead zero is chosen the lead pole is computed from $p = \sigma_0 + \frac{\omega_0}{\tan(\theta_p)}$ where $\theta_p = \theta_z - \theta_c$.

Procedures for other compensators can be found in previous work[9.10] A complete list of the compensators included in the GUI is provided in Table 2.

In order to make the discussion of the GUI simpler, the GUI can be divided into 3 regions. The region on the left side of the GUI displays the uncompensated and compensated root loci. The right side of the GUI displays the uncompensated and compensated Bode magnitude and Bode phase plots. The central region of the GUI is where the user enters the system specifications. The user can select the compensator type, system plant, design point, ϕ_0 , and compensator zeros (if applicable based on the compensator chosen). When the **Compensator** drop-down box is selected, the GUI gives the user the choice of selecting between the compensators listed in Table 2. For the PID and PI-Lead compensators, the user has the ability to select different compensator zeros because the unknown parameters in the compensator outnumber the constraints.

The system specifications, which may be determined from the rise time (T_r), settling time (T_s), peak time (T_p), and percent overshoot ($P.O.$), are entered in terms of the **Design Point** and ϕ_0 (labeled Phi_d on the GUI) input fields. The **Update** button is pressed to execute the GUI. The GUI then plots the root locus and Bode plots in their respective areas. The **Analysis** option in the toolbar allows the user to plot the open-loop Bode response, closed-loop step response, and closed-loop Bode response.

Example using Root Locus Method

In this example, the compensator for an antenna-angle tracker will be designed. The $G_p(s)$ in Figure 1 is represented by the transfer function $G_p(s) = \frac{1}{s(10s+1)}$. [12] The system input $U(s)$ is the voltage applied to the servo motor. The system output $Y(s)$ is the angular position of the antenna. The design

Table 2. Compensator Types.				
Compensator	Description	Transfer Function	Transient Response	Steady-State Error (e_{ss})
PD	Proportional-Derivative	$G_c = (s + z)$	Improve	-
Lead	Lead	$G_c = \frac{(s + z)}{(s + p)}$	Improve	-
PI	Proportional-Integral	$G_c = \frac{(s + z)}{s}$	Degrade	Improve
Lag	Lag	$G_c = \frac{(s + z)}{(s + p)}$	Degrade	Improve
PID*	Proportional-Integral-Derivative	$G_c = \frac{(s + z)^2}{s}$	Improve	Improve
PID**	Proportional-Integral-Derivative	$G_c = \frac{(s + z_1)(s + z_2)}{s}$	Improve	Improve
PI-Lead*	Proportional-Integral-Lead	$G_c = \frac{(s + z)^2}{s(s + p)}$	Degrade	Improve
PI-Lead**	Proportional-Integral-Lead	$G_c = \frac{(s + z_1)(s + z_2)}{s(s + p)}$	Degrade	Improve
*colocated zeros **noncolocated zeros $z < p$ for lead compensator $z > p$ for lag compensator				

specifications for a step input are the following: percent overshoot less than 16%, settling time less than 1.66 seconds, and a steady-state error of zero. From basic second order approximations, one design point that satisfies these constraints is

$$s_0 = -\sigma_0 \pm j\omega_0 = -2.41 \pm j4.13.$$

The first step is to plot the uncompensated plant information. The uncompensated root locus is shown in Figure 4. The second step is to determine if a compensator is necessary. The root locus is to the right of the design point s_0 , therefore, a compensator is needed. Because the

plant is a Type 1 system, the steady state error criterion is automatically met for a step input provided that the closed-loop system is stable. A lead compensator is chosen because it pulls the root locus left and improves the transient response of the system. The GUI calculates the compensator angle θ_c . The compensator poles and zeros must be selected. Figure 2 shows the relationship between the lead compensator pole, zero, pole angle (θ_p), and zero angle (θ_z). For a lead compensator, the zero (z_{lead}) must be selected such that it is smaller than the zero of a PD compensator (z_{PD}). Using a PD compensator in the GUI, we get $z_{PD} = 4.84$.

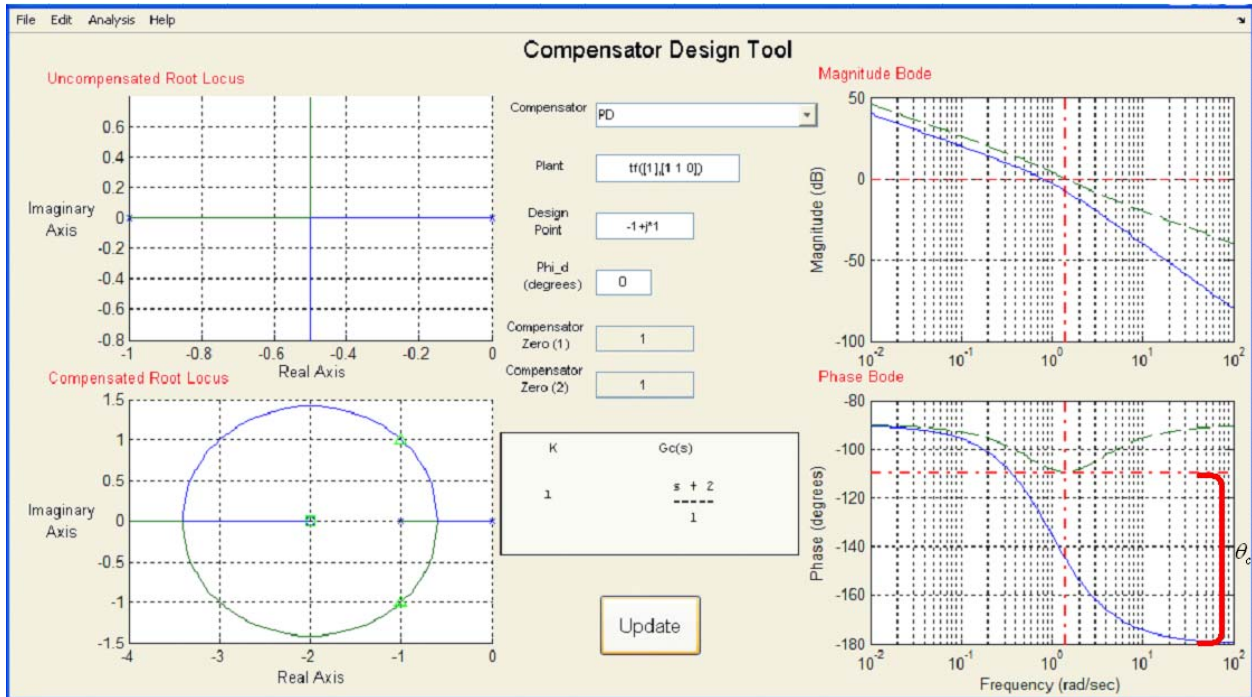


Fig. 3. Graphical User Interface (GUI) in Matlab.

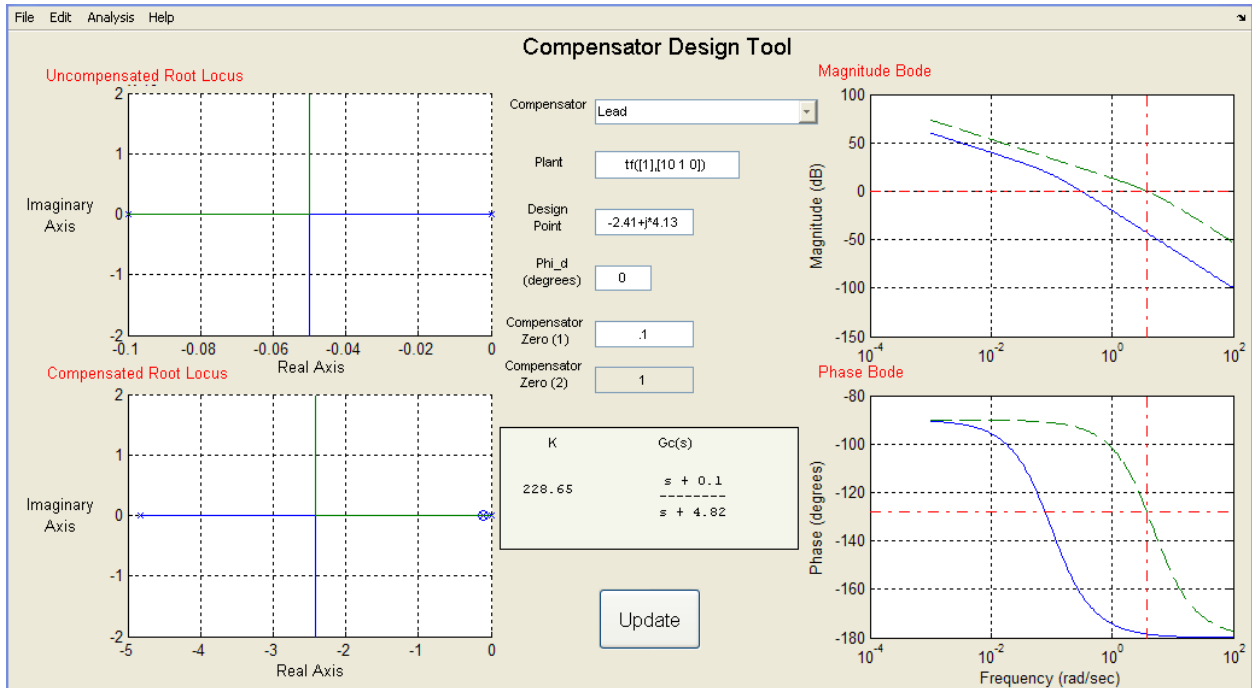


Fig. 4. Example with $s_0 = -2.41 \pm j4.13$, $z=0.1$, and $\phi_0 = 0^\circ$.

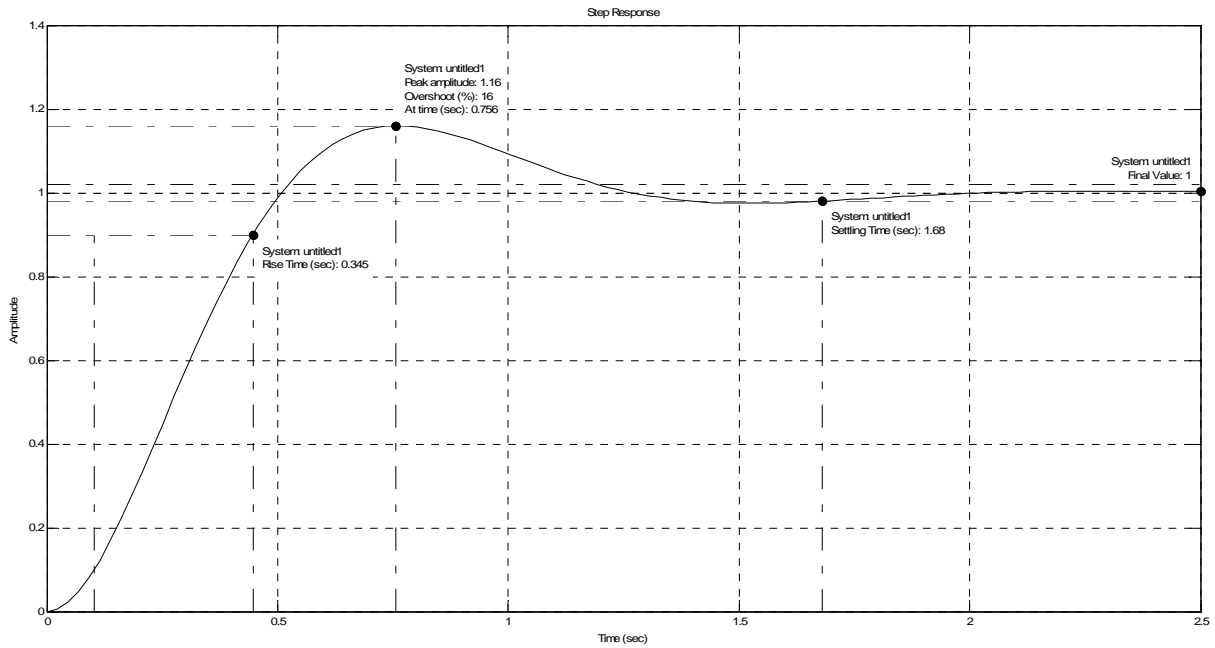


Fig. 5. Closed-loop step response with $s_0 = -2.41 \pm j4.13$, $z=0.1$, and $\phi_0 = 0^\circ$.

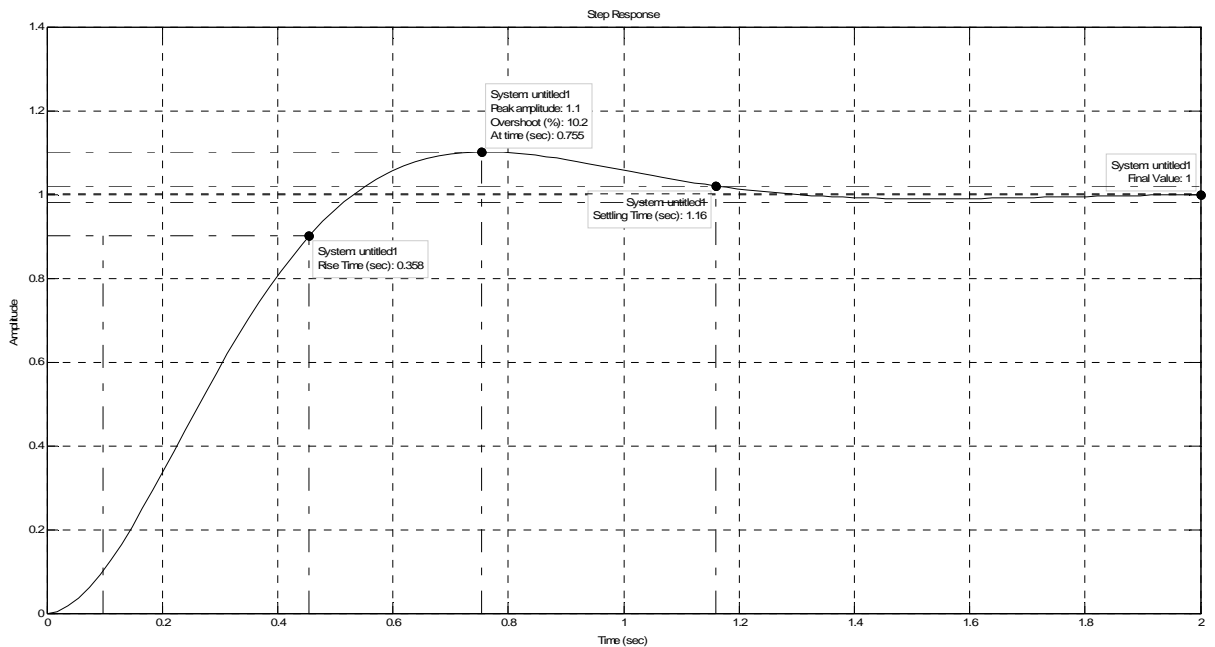


Fig. 6. Closed-loop step response with $s_0 = -3 \pm j4.13$, $z=0.1$, and $\phi_0 = 0^\circ$.

We will choose $z_{lead} = 0.1$ in order to ensure that the design point is as dominant as possible. The value of the θ_z is then calculated by $\theta_z = \tan^{-1}\left(\frac{\omega_0}{z_{lead} - \sigma_0}\right)$. The value of p can be obtained by $\theta_c = \theta_z - \theta_p$ and

$$p = \sigma_d + \frac{\omega_d}{\tan(\theta_p)}$$

The GUI then calculates the value of the control gain K from the magnitude criterion in (3).

Next we evaluate, simulate, and (if necessary) redesign. From Figure 5, the $P.O.=16\%$ and

$T_s=1.68$ seconds. The design specification for $P.O.$ was met. However, the design specification for T_s was not met. Using our second order approximations, a new design point was chosen as $s_0 = -3 \pm j4.13$. The real part of s_0 , σ_0 , was increased in order to improve T_s . Figure 6 shows the closed-loop step response for the new design point. The new design point does meet the design specifications with $P.O.=10.2\%$ and $T_s=1.16$ seconds. The new compensator is

$$G_c(s) = 228.65 \frac{(s+0.1)}{(s+4.82)}.$$

Example using Bode Method

We will repeat the example using the Bode method with the same plant and design specifications the same as before. The relationship between the damping ratio and the PM in (1) can be approximated by

$$(1) \quad PM = \begin{cases} 100^\circ \zeta & 0 \leq \zeta \leq 0.6 \\ 45^\circ \zeta + 33^\circ & 0.6 \leq \zeta \leq 1.0 \end{cases}$$

From (9) and (2), one design point that satisfies the specifications is $s_0 = -\sigma_0 \pm j\omega_0 = j\omega_{gc} = j2.02$ with $\phi_0 = PM = 50^\circ$.

The first step is to plot the uncompensated Bode plot. The GUI plots the uncompensated plot (in blue) in Figure 7. The uncompensated Bode phase response cannot satisfy the PM requirement for $\omega_{gc} \geq 2.02$. This leads us to the conclusion that a compensator is needed. A lead compensator is selected in order to improve the transient response.

As in the root locus example, θ_c is calculated and a lead compensator zero is chosen with $z_{lead} = 0.1$. After calculating the pole location, the GUI calculates the value of the control gain K from the magnitude criterion in (3). The final step is to evaluate, simulate, and (if necessary)

redesign. From Figure 8, the $P.O.=18.1\%$ and $T_s=3.27$ seconds. Neither one of the design specifications were met. Using (9) and (2), ω_{gc} was increased to 3 rad/sec and the PM to 60° . Figure 9 shows the closed-loop step response for the new design point. With the new design point, the system does meet the design specifications with the $P.O.=8.77\%$ and $T_s=1.41$ seconds. The new compensator is

$$G_c(s) = 180 \frac{(s+0.1)}{(s+5.1962)}.$$

Ball and Beam Laboratory Exercise

To demonstrate how the GUI can be used in the laboratory, consider the following project involving the ball and beam apparatus shown in Figure 10. A control architecture with two loops can be used to control the position of the ball on the beam. First, an inner loop is designed to control the motor position and consequently the angle of the beam. The outer loop is then designed to control the position of the beam. The outer loop is often designed assuming that the inner loop is arbitrarily fast.

In this experiment, the students were asked to study the robustness of their outer loop control design to the unmodeled dynamics of the inner loop. To accomplish this, the students needed to develop several designs for the inner loop control, each with a different speed of response. Normally, this would have required a fairly lengthy and tedious design process. However, with the GUI, the students were able to quickly design several inner loop controls, implement them on the ball and beam apparatus, and study the robustness properties that were the main goal of the exercise.

Conclusion

The GUI can be a very effective teaching tool. It enables the user to see how both the root locus and the Bode plots can be used for control system design. It is easy to demonstrate whether a particular compensator is needed. It can also be used to verify whether the design

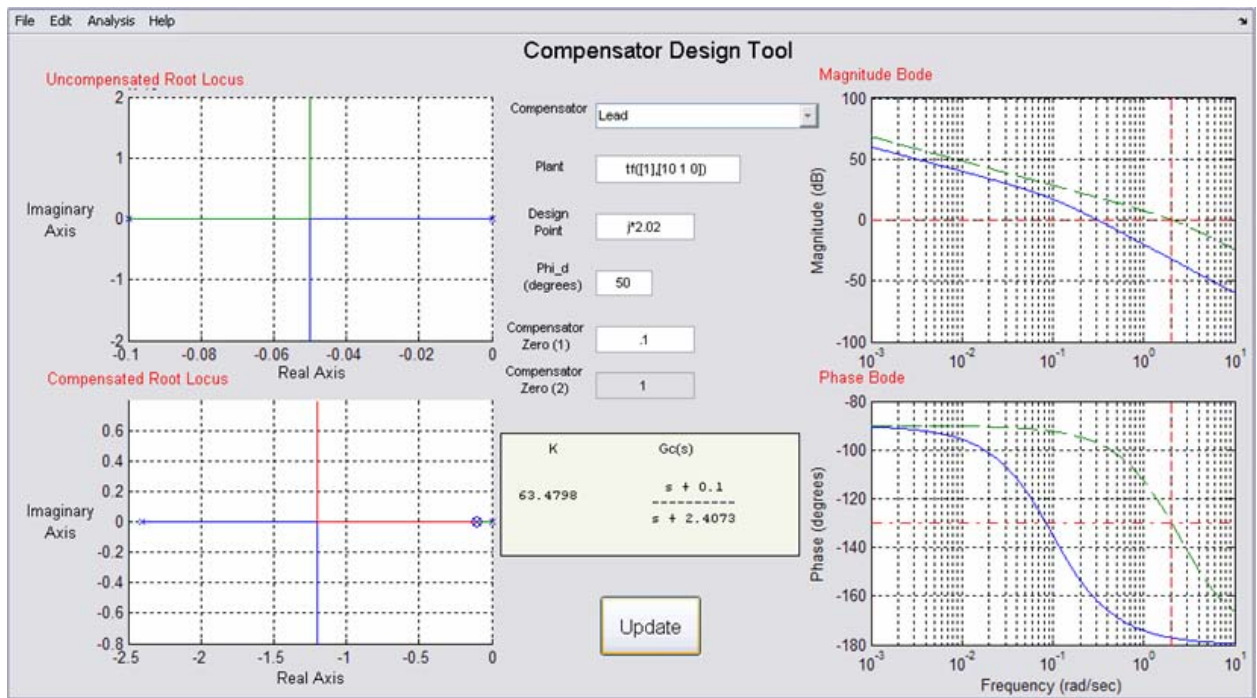


Fig. 7. Example GUI with $s_0 = j\omega_{gc} = j2.02$, $z=0.1$, and $\phi_0 = 50^\circ$.

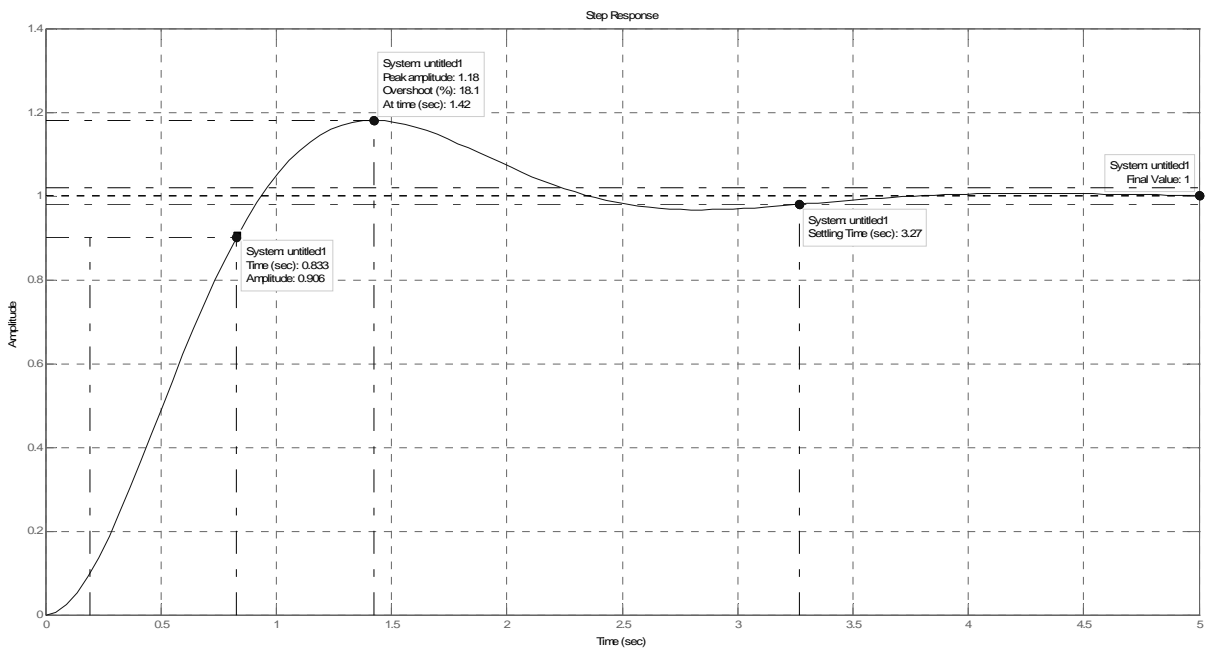


Fig. 8. Closed-loop step response with $s_0 = j\omega_{gc} = j2.02$, $z=0.1$, and $\phi_0 = 50^\circ$.

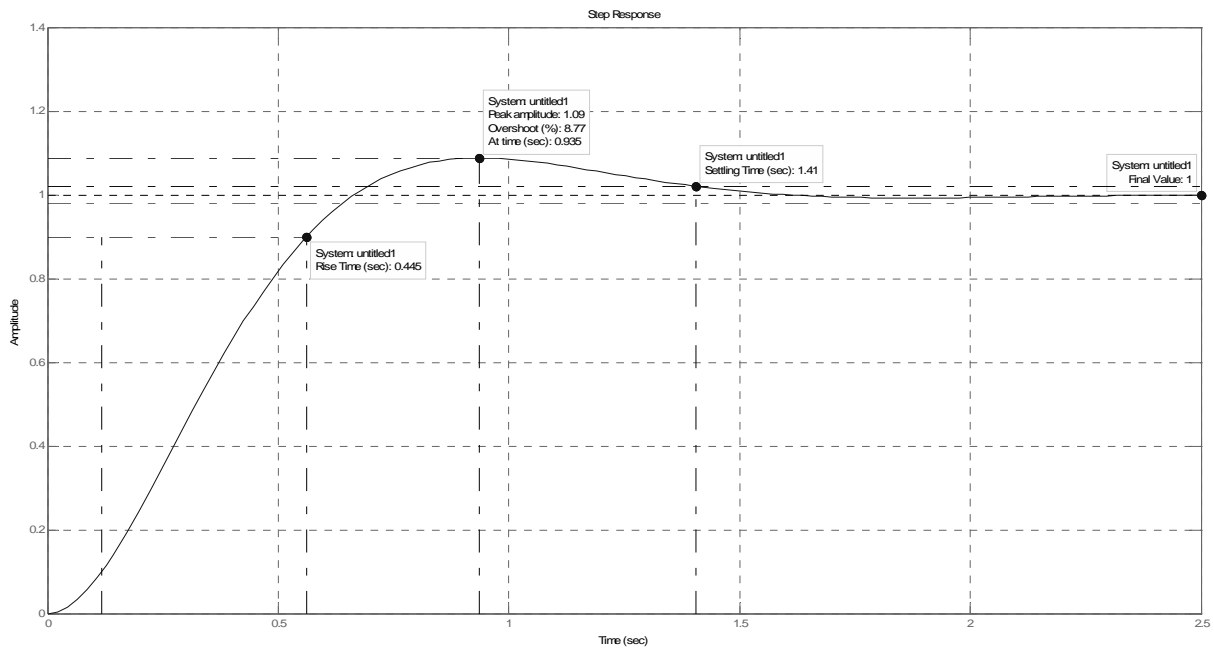


Fig. 9. Closed-loop step response with $s_0 = j\omega_{gc} = j3$, $z=0.1$, and $\varphi_0 = 60^\circ$

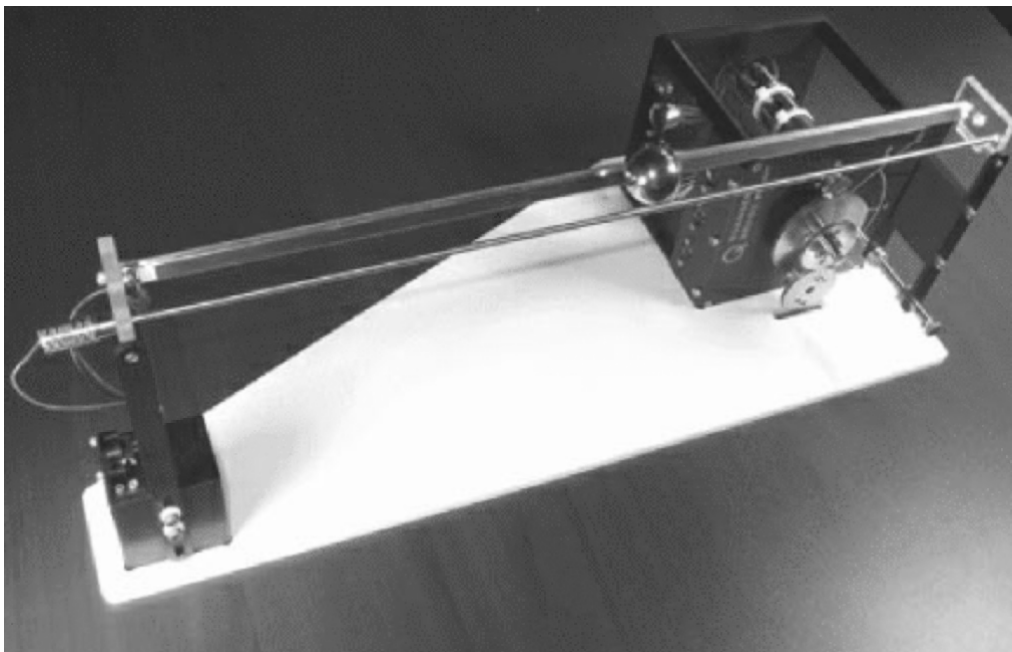


Fig. 10. Ball and beam apparatus.

specifications are met. The GUI also allows the user to pick either one of the compensator design methods in the unified approach. If the root locus method is chosen, $s_0 = s_d$ and ϕ_0 is 0. If the Bode method is chosen, $s_0 = j\omega_{gc}$ and $\phi_0 = PM$.

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