DESIGN OF A MULTI-MODE FINITE-DIFFERENCE HEAT TRANSFER PROJECT

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Abstract

The development of a comprehensive finite-difference project at the end of a heat transfer curriculum is described. The problem requires evaluation of the school's football field turf heating system, incorporates all of the major heat transfer modes (convection, conduction, and radiation), and requires students to investigate both steady state and transient versions of the problem, with comparison to analytical solutions when available. The problem is solved using the finite-difference method (FDM) and an Excel™ spreadsheet with Visual Basic for Applications™ (VBA) programming to facilitate program execution beyond normal spreadsheet capabilities. The project also requires students to conduct a design analysis for environmental and/or system changes, subject to approval by the instructor; suggested topics for this design project are offered. The project is designed so that it is easy for students to understand, and recommendations are offered regarding project design and submission which facilitate grading of student work. While the specific application described herein is to the school's football field, the same approach may be employed in many steady state and transient heat transfer problems—in fact, students have employed the modeling and programming techniques learned in this project to other courses, including their Senior Capstone projects. Results of a student opinion survey, anecdotal data, and performance on the heat transfer portion of the Fundamentals of Engineering examination data are presented.

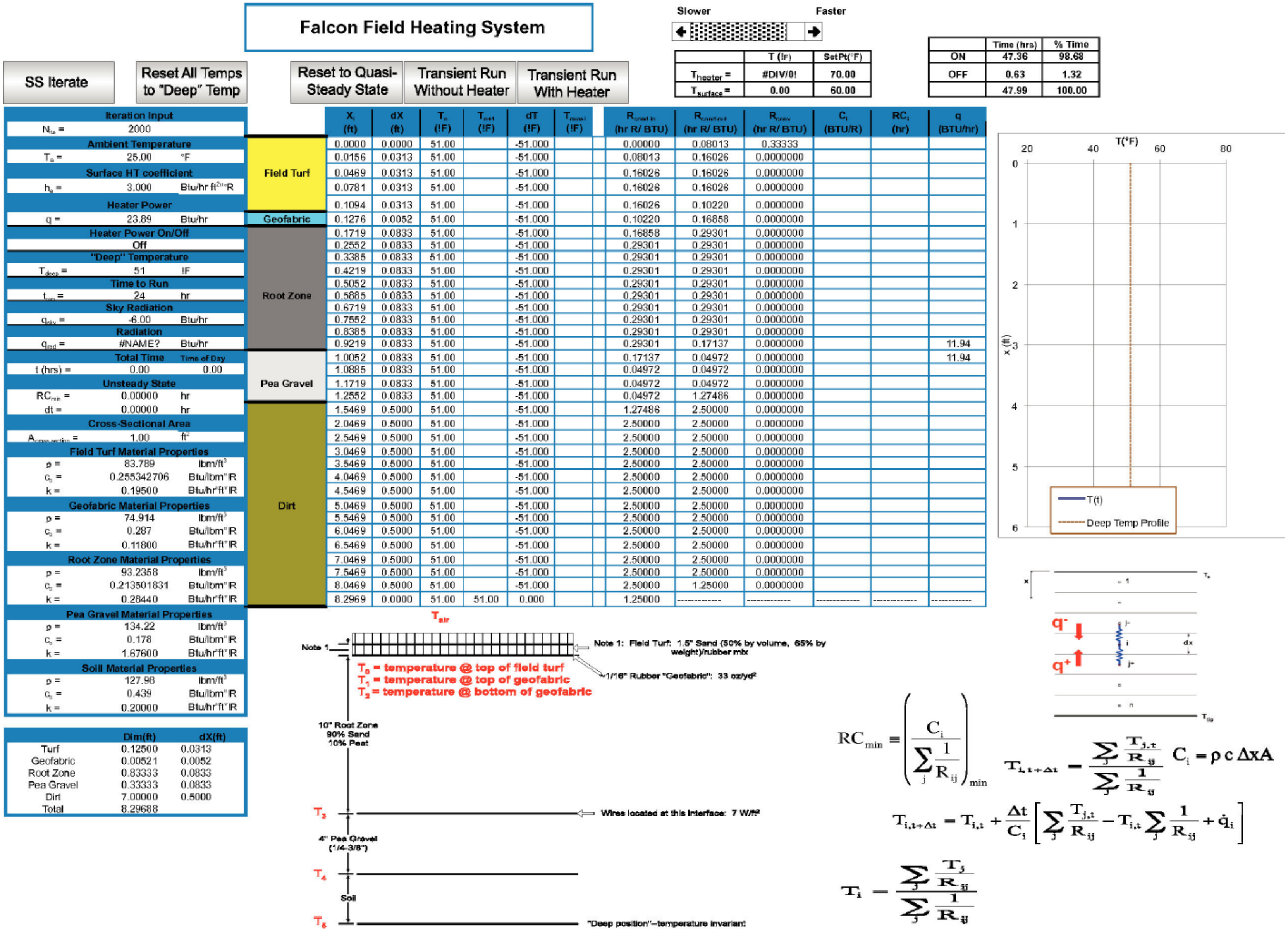
Considerations in Designing a FDM Project

When designing a problem appropriate for completion in a certain number of class meetings, educational and institutional constraints must be considered. As opposed to the finite-element method (FEM), the finite-difference method (FDM) does not have a steep learning curve and was therefore ideal to provide students with a tool which would allow analysis of many realistic heat transfer processes. Additionally, it was desired that whatever computational code was employed for implementation of the FDM should be readily available and of low cost; Microsoft Excel™ proved to be ideally suited in this regard, especially since all Microsoft products include the Visual Basic for Applications™ (VBA) programming language. VBA possesses more extensive computational capability than available in a basic spreadsheet; as juniors, students in the mechanical engineering curriculum at USAFA already receive a block of instruction in VBA object-oriented programming [1], making Excel™ an easy choice for FDM implementation. The project assigned had to include all modes of heat transfer, and needed to involve scenarios which require analysis under both steady state and transient conditions.

Being an undergraduate institution, no teaching assistants are assigned at USAFA, so that the instructor must grade all work submitted; ease of grading was, therefore, an important consideration. A fine line had to be walked in ensuring not only the required degree of difficulty, but also allowing that mistakes be easily detected. Figure 1 is a screen shot of the template provided to the students, with all material properties and dimensions depicted. Not only were students required to provide all equations in the blank cells of this template, but they were required to do a certain amount of macro recording and assignment of macros to the control buttons shown. Iteration speed for the steady state problem and time stepping speed in the transient problem were controlled by varying the screen refresh rate using the slide bar shown.

To gauge their progress, formula *results* (rather than formulae) for certain cells were provided to students for comparison with their spreadsheet results. Additionally, two versions of the steady state spreadsheet and two versions of the transient spreadsheet were required turn-in items; each required grids and row and column headings to be displayed. The first version merely displayed the calculation results (i.e., “numbers”), while the second displayed the embedded formulae. Maximum use was made of named (i.e., absolute reference) cells so that formulae entered by students could, for the most part, be readily corrected; indeed, students also found this to be a boon to troubleshooting problems while setting up their spreadsheets.

A common analogy used to teach conduction, convection, and radiation heat transfer is that of the electrical circuit, especially with regard to heat transfer (or electrical current flow) through a resistance; the concept of a thermal capacitance (its analog being the electrical capacitor) is less likely to be taught but is crucial to the understanding of transient heat transfer processes. Unfortunately, when the FDM is taught for use in heat transfer calculations, this entire analogy is frequently abandoned. Baughn [1] recognized this, and proposed a method whereby this analogy could be readily taught and implemented using spreadsheets. At USAFA, following completion of the last heat transfer topic (radiation) in the fall term of the senior year of the mechanical engineering curriculum, Baughn’s method is used to introduce the FDM in one dimension, applied to both steady state and transient problems in a 7-lesson block over three weeks. Simple, closed-form problems (such as the steady heat transfer through a slab, and transient cooling of the same slab in air) are first solved analytically, and then set up for solution via the FDM; agreement (and any disagreements!) are noted, and confidence in the FDM is bolstered. More open-ended problems are then tackled, noting trends in the solution to different boundary and initial conditions. Generally, the steady state and transient problems each count as a major homework assignment with students working in teams of two, and with no collaboration (to include comparison of answers!) allowed between groups; students are allowed (and encouraged) to meet with the instructor to discuss each problem as they work through each. The steady state project followed three simpler homework problems solved via the FDM; it was due two lessons later (one of which was dedicated to work on the steady state project in class). During that time, instruction continued in the transient method, to include one transient FDM problem, one class period devoted to introduction to their transient project, and another devoted to in-class work on the project. The transient project was due three lessons after it was assigned. A project of this nature would follow the basic coverage of conduction, convection, and radiation, and would occupy the last three weeks of a 3-hour semester heat transfer course.



	T (!F)	SetPt(°F)
T_{heater} =	#DIV/0!	70.00
$T_{surface}$ =	0.00	60.00

	Time (hrs)	% Time
ON	47.36	98.68
OFF	0.63	1.32
	47.99	100.00

Iteration Input		
N_{its} =	2000	
Ambient Temperature		
T_a =	25.00	°F
Surface HT coefficient		
h_a =	3.000	Btu/hr ft^{2}°R
Heater Power		
q =	23.89	Btu/hr
Heater Power On/Off		
	Off	
"Deep" Temperature		
T_{deep} =	51	!F
Time to Run		
t_{run} =	24	hr
Sky Radiation		
q_{sky} =	-6.00	Btu/hr
Radiation		
q_{rad} =	#NAME?	Btu/hr
Total Time		**Time of Day**
t (hrs) =	0.00	0.00
Unsteady State		
RC_{min} =	0.00000	hr
dt =	0.00000	hr
Cross-Sectional Area		
$A_{cross\ section}$ =	1.00	ft^2
Field Turf Material Properties		
ρ =	83.789	lbm/ft^3
c_p =	0.255342706	Btu/lbm"!R
k =	0.19500	Btu/hr'ft'!R
Geofabric Material Properties		
ρ =	74.914	lbm/ft^3
c_p =	0.287	Btu/lbm"!R
k =	0.11800	Btu/hr'ft'!R
Root Zone Material Properties		
ρ =	93.2358	lbm/ft^3
c_p =	0.213501831	Btu/lbm"!R
k =	0.28440	Btu/hr'ft'!R
Pea Gravel Material Properties		
ρ =	134.22	lbm/ft^3
c_p =	0.178	Btu/lbm"!R
k =	1.67600	Btu/hr'ft'!R
Soil Material Properties		
ρ =	127.98	lbm/ft^3
c_p =	0.439	Btu/lbm"!R
k =	0.20000	Btu/hr'ft'!R

	Dim(ft)	dX(ft)
Turf	0.12500	0.0313
Geofabric	0.00521	0.0052
Root Zone	0.83333	0.0833
Pea Gravel	0.33333	0.0833
Dirt	7.00000	0.5000
Total	8.29688	

	X_i (ft)	dX (ft)	T_n (!F)	T_{n+1} (!F)	dT (!F)	T_{quasi} (!F)	$R_{cond\ in}$ (hr R/ BTU)	$R_{cond\ out}$ (hr R/ BTU)	R_{conv} (hr R/ BTU)	C_i (BTU/R)	RC_i (hr)	q (BTU/hr)
Field Turf	0.0000	0.0000	51.00		-51.000		0.00000	0.08013	0.33333			
	0.0156	0.0313	51.00		-51.000		0.08013	0.16026	0.0000000			
	0.0469	0.0313	51.00		-51.000		0.16026	0.16026	0.0000000			
	0.0781	0.0313	51.00		-51.000		0.16026	0.16026	0.0000000			
	0.1094	0.0313	51.00		-51.000		0.16026	0.10220	0.0000000			
Geofabric	0.1276	0.0052	51.00		-51.000		0.10220	0.16858	0.0000000			
Root Zone	0.1719	0.0833	51.00		-51.000		0.16858	0.29301	0.0000000			
	0.2552	0.0833	51.00		-51.000		0.29301	0.29301	0.0000000			
	0.3385	0.0833	51.00		-51.000		0.29301	0.29301	0.0000000			
	0.4219	0.0833	51.00		-51.000		0.29301	0.29301	0.0000000			
	0.5052	0.0833	51.00		-51.000		0.29301	0.29301	0.0000000			
	0.5885	0.0833	51.00		-51.000		0.29301	0.29301	0.0000000			
	0.6719	0.0833	51.00		-51.000		0.29301	0.29301	0.0000000			
	0.7552	0.0833	51.00		-51.000		0.29301	0.29301	0.0000000			
	0.8385	0.0833	51.00		-51.000		0.29301	0.29301	0.0000000			
	0.9219	0.0833	51.00		-51.000		0.29301	0.17137	0.0000000			11.94
Pea Gravel	1.0052	0.0833	51.00		-51.000		0.17137	0.04972	0.0000000			11.94
	1.0885	0.0833	51.00		-51.000		0.04972	0.04972	0.0000000			
	1.1719	0.0833	51.00		-51.000		0.04972	0.04972	0.0000000			
	1.2552	0.0833	51.00		-51.000		0.04972	1.27486	0.0000000			
Dirt	1.5469	0.5000	51.00		-51.000		1.27486	2.50000	0.0000000			
	2.0469	0.5000	51.00		-51.000		2.50000	2.50000	0.0000000			
	2.5469	0.5000	51.00		-51.000		2.50000	2.50000	0.0000000			
	3.0469	0.5000	51.00		-51.000		2.50000	2.50000	0.0000000			
	3.5469	0.5000	51.00		-51.000		2.50000	2.50000	0.0000000			
	4.0469	0.5000	51.00		-51.000		2.50000	2.50000	0.0000000			
	4.5469	0.5000	51.00		-51.000		2.50000	2.50000	0.0000000			
	5.0469	0.5000	51.00		-51.000		2.50000	2.50000	0.0000000			
	5.5469	0.5000	51.00		-51.000		2.50000	2.50000	0.0000000			
	6.0469	0.5000	51.00		-51.000		2.50000	2.50000	0.0000000			
	6.5469	0.5000	51.00		-51.000		2.50000	2.50000	0.0000000			
	7.0469	0.5000	51.00		-51.000		2.50000	2.50000	0.0000000			
	7.5469	0.5000	51.00		-51.000		2.50000	2.50000	0.0000000			
	8.0469	0.5000	51.00		-51.000		2.50000	1.25000	0.0000000			
	8.2969	0.0000	51.00	51.00	0.000		1.25000	------------	------------	------------	------------	------------

$$RC_{min} \equiv \left(\frac{C_i}{\sum_j \frac{1}{R_{ij}}} \right)_{min}$$

$$T_{i,t+\Delta t} = \frac{\sum_j \frac{T_{j,t}}{R_{ij}}}{\sum_j \frac{1}{R_{ij}}} \qquad C_i = \rho\, c\, \Delta x A$$

$$T_{i,t+\Delta t} = T_{i,t} + \frac{\Delta t}{C_i}\left[\sum_j \frac{T_{j,t}}{R_{ij}} - T_{i,t} \sum_j \frac{1}{R_{ij}} + \dot{q}_i \right]$$

$$T_i = \frac{\sum_j \frac{T_j}{R_{ij}}}{\sum_j \frac{1}{R_{ij}}}$$

Figure 1: Blank worksheet FDM template as distributed to students.

Background

Falcon Stadium is the location where the United States Air Force Academy (USAFA) holds its home football games. Being an outdoor stadium in Colorado subjected to the quickly changing weather associated with the Front Range of the Rocky Mountains, games in the latter part of the season are frequently played at or near freezing temperatures and on occasion, snow. To circumvent the problem of hard playing fields due to the cold temperatures, many colleges and professional football organizations install heating systems underneath their fields to maintain a warmer field; this allows cleats to dig in to the dirt, instead of riding on top of the field, with a concomitant reduction in injuries. (N.B.: While the example provided herein is presented in British units, problems may be posed and solved in any unit system desired, depending on individual instructor or institutional preferences).

The USAFA installed a field heating system while the stadium was being built in 1965. Field heating systems then typically employed electric resistance heating which, based on the state of technology and the cost of electricity at the time, made the most sense for Falcon Stadium. Little data exist regarding the field heating system and used until major renovation in 1997. During the renovation, the entire field was excavated and replaced down to the indigenous soil, at which time the field heating system also underwent a total renovation. While consideration was given to switching the heating system from an electrical resistance type to one which employed a pumped fluid, budgetary constraints dictated that the existing electrical system would be modified since the required supporting infrastructure was already in place. Approximately 26.5 miles of wire was laid down over the field, creating three heating zones, each running the length of the field. An important change from the previous heating system was to space the wires six inches apart, as opposed to the previous twelve-inch spacing. The only requirement given to the field heating system contractor was to maintain a 60° Fahrenheit temperature at the heater and a 50° Fahrenheit temperature at the surface of the field, mainly to prevent the field from freezing.

In 2005, the field underwent its most recent renovation, changing the sod to an artificial turf. FieldTurf™ was chosen since it reduced field maintenance costs and resulted in a much more professional appearance when games were televised. During this renovation, only the sod was replaced with FieldTurf™. Figure 2 shows the current cross sectional make-up of the Falcon field turf heating system.

The 1.5-inch layer of FieldTurf™ constitutes the top layer of the field, the makeup of which includes plastic “grass” blades, sand, and rubber pellets; proportions of the FieldTurf™ mixture are listed in Figure 2. The rubber pellets are what give this layer its sod-like softness, while the sand enhances traction. Beneath the field turf is a layer of 1/16-inch rubber “geofabric,” to which the blades of artificial grass are attached. Immediately beneath the geofabric is ten inches of root zone, which is a remnant of the sod field; in order for the old sod to grow and take hold at Falcon stadium, the field root zone needed to be of the same composition as the root zone at which the sod was originally grown, hence the composition of the root zone shown in Figure 2. The heating system wires, situated in between the root zone and pea gravel, deliver 7 W/ft^2. The pea gravel facilitates drainage, as there are collection troughs to channel the moisture away from the field; it is comprised of ¼ - 3/8 inch in diameter pea gravel, and is thought to be Chena River gravel. Finally, beneath the pea gravel lies the indigenous soil.

Much of the experience running the field heating system has been from trial and error. When the field had a sod surface, it would typically be run from the second or third week

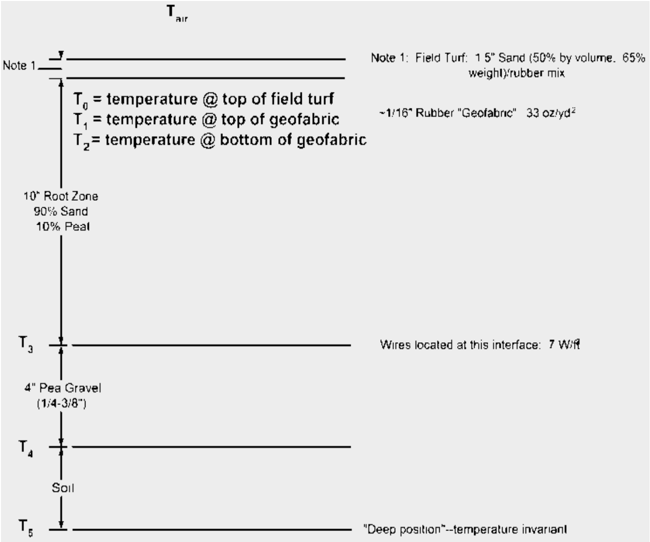


Figure 2: Cross-section of field and location of heater.

of October until the last game of the season. Once the last game had been played, the temperature of the field would be lowered by 2-3°F per week, until the heater was no longer being operated. After the 2005 field renovation,a significant change in performance of the heating system was noticed; it then took quite a bit longer to heat the field and melt the snow on the surface. When asked about any testing of the heating system to see if the system was able to meet the requirements laid out to the contractors, stadium staff were not aware of any and could not find any documentation to that end. A trip to the 10th Civil Engineering Squadron returned similar results as their records on the 1997 renovation had passed their retention period and had since been destroyed.

A student independent study project during the Spring 2009 term was conducted to construct a finite-difference model of the problem, and was continued by two other students during the Fall 2009 term. The principal purpose of this work was to provide stadium staff with recommendations on how to intelligently utilize the system, based on the current artificial turf installation; results are provided in [3]. Secondarily, a simplified version of the problem was perfect for implementation in the mechanical engineering major heat transfer curriculum, after students had been exposed to all conduction, convection, and radiation modes of heat transfer, and the rudimentary aspects of the finite-difference method; the current paper is concerned with presenting how this project was implemented as part of a heat transfer course. Previous projects assigned included analysis and design aircraft Pitot-tube and wing anti-icing systems, and also aircraft braking systems [2].

Physical Model

Several assumptions were made to simplify the football field heating problem to allow it to be analyzed by students in the heat transfer portion of the thermal fluids engineering curriculum. These included:

A. The temperature at a 20-foot depth in the area surrounding the field is 51°F year-round, and seasonal variations are minimal at even shallower depths [4]. For this problem, it was assumed that the temperature at a depth between 8 and 9 feet is 51°F.

B. The only modes of heat transfer which were considered were conduction through the ground, convection with the ambient air, solar insolation, and losses due to sky radiation.

   1. Each day in the simulation was assumed "identical," with the same variation of ambient temperature and insolation. While only one home game has been played in December in the last 13 years, the temperature profile for this month proved to be the "worst case" scenario for the football season. Hourly temperature averages for December 2005 logs at the Colorado Springs Airport were used to construct curve fits which allowed ambient temperature interpolation for any desired time.

2. Insolation data were obtained from a University of Oregon online program [5]. These data were already corrected for the effects of altitude and an average value of local temperature; prior to incorporation into the program, they were further corrected for an assumed cloud cover and surface absorptivity. A sinusoidal variation between zero and the midday peak insolation intensity was assumed between sunrise and sunset; the peak temperature and sunrise/sunset times for the middle of the month were utilized.

3. Sky radiation losses and the convective heat transfer coefficient were each taken as constant [6].

4. Heat transfer was considered to be strictly one-dimensional.

5. When activated, the resistance heater provides a power flux of 7W/ft^2.

6. A desired minimum playing field surface temperature of 50°F was specified, and temperature in the vicinity of the heater was not to exceed 60°F.

7. The effects of moisture changes in the ground and associated latent heat transfer were neglected, as were the effects of precipitation (snow, rain, etc.).

8. All property values were assumed invariant.

9. Each component material was assumed isotropic.

10. The field surface was required to be at the specified temperature starting at noon on game day and to be sustained for another four hours.

11. Properties of composite materials (i.e., sand/rubber or sand/peat) were evaluated in advance and provided in the spreadsheet template.

The only "tricky" item not covered in the conduction module of the earlier thermal fluids engineering courses was in evaluating the resistance between adjoining nodes where each node was centered in an element of a different material. Students were guided in how to calculate this by considering that the overall resistance between these nodes was the sum of half of each element's total resistance.

Steady State

The Gauss-Seidel relaxation method was employed, and involved repeated solution of the temperature at each element or node in terms of the temperatures of neighboring elements; convergence to the steady state solution was eventually achieved. The basic equation given by Baughn [2] for the temperature of the i-th node was of the form (see Figure 3 for notation)

$$T_i = \frac{\dot{q}_i + \sum_j \frac{T_j}{R_{ij}}}{\sum_j \frac{1}{R_{ij}}} \qquad (1)$$

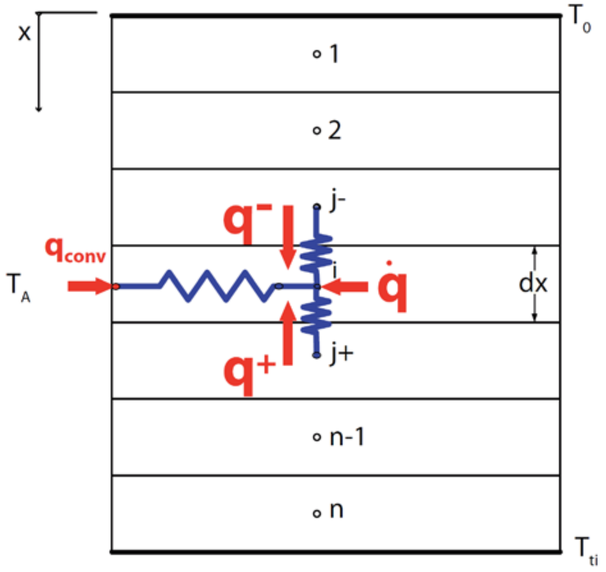


Figure 3: Steady-state nodal layout, showing node of interest (i^{th} node) surrounding nodes (j^- and j^+), and heat flow sign conventions.

The summations are conducted across the j neighboring nodes, with R_{ij} denoting the resistance between the node in question (i) and each surrounding node (j); $\dot{q}_i$ indicates the volumetric heat generation (if any) associated with the i-th node, typically one which adjoins the heating element. Note that the use of thermal resistances (both conductive and convective) is consistent with students' exposure to the electrical circuit analogy in their study of heat transfer.

To build student confidence in the FDM as applied to this problem, they were first tasked to analyze the heat transfer through the composite medium described above, using constant air and "deep" temperatures. Prior to setting up the FDM, students were to solve the problem analytically, so that a basis existed for checking various items from the FDM: temperature profile, heat transfer rate, resistance of each layer, temperature profile, etc. After solving the problem, students had to plot the temperature profile after various numbers of iterations to provide a graphic depiction of the convergence associated with Gauss-Seidel iteration. A critical comparison of the analytical and FDM results was required.

Using the same model, the heater was then introduced, and distributed across the elements immediately above and below the root zone/pea gravel interface; again, students were tasked with solving the problem analytically so that comparisons could be made. The analytical problem was one with which the students had not yet been confronted, and required a series of questions designed to walk them through the problem. Specifically, they were to find the heater temperature (T_3) and the resistances above (R_{upper}) and below (R_{lower}) the heaters (actually, these two resistances were already available from the case where the heaters had not been activated), and then to imagine the problem in terms of the electrical analog, with the heater power dividing into two paths: one upward, the other downward, resulting in the following equation (temperature locations keyed to Figure 2):

$$q_{upper} = \frac{T_3 - T_{air}}{R_{upper}} \qquad q_{lower} = \frac{T_3 - T_5}{R_{lower}} \tag{2}$$

$$\begin{aligned} q_{electrical} &= q_{upper} + q_{lower} \\ &= \frac{T_3 - T_{air}}{R_{upper}} + \frac{T_3 - T_5}{R_{lower}} \\ &= \frac{R_{lower}(T_3 - T_{air}) + R_{upper}(T_3 - T_5)}{R_{lower}R_{upper}} \end{aligned} \tag{3}$$

All items in these equations except T_3 were known values. Once T_3 was obtained, the heat transfer rates in the upper and lower directions (q_{upper} and q_{lower}, respectively) could be found; students were asked to comment on the magnitudes of these two heat transfer rates relative to the resistance in their respective directions. Students were then also told to note that these heat transfer rates were superposed on the heat transfer they had originally calculated for the situation without the heater activated. Once obtained, these results were compared with those resulting from the FDM run with a large number of iterations with the heater activated.

Transient

The implicit method of solving for temperature distributions involves matrix inversion, and does not lend itself well to implementation in a spreadsheet. Consequently, the explicit method is employed here, and involves calculating the temperature of the i-th node at the next time step (i.e., at time t + Δt) using the current (i.e., at time t) value of temperatures; Baughn [2] gives the temperature of the i-th node as

$$T_{i,t+\Delta t} = T_{i,t} + \frac{\Delta t}{C_i}\left[\sum_j \frac{T_{j,t}}{R_{ij}} - T_{i,t}\sum_j \frac{1}{R_{ij}} + \dot{q}_i\right] \tag{4}$$

Notation for this equation is the same as for the steady state formulation, with the addition of the thermal capacitance of the i-th node calculated as

$$C_i = Vc_i \tag{5}$$

where V_i is the element's volume and c_i its specific heat. Figure 4 illustrates the incorporation of convection. For surface nodes which have no volume (and consequently no thermal capacitance), the equation is very similar to that for steady state iteration:

$$T_{i,t+\Delta t} = \frac{\dot{q}_i + \sum_j \frac{T_{j,t}}{R_{ij}}}{\sum_j \frac{1}{R_{ij}}} \tag{6}$$

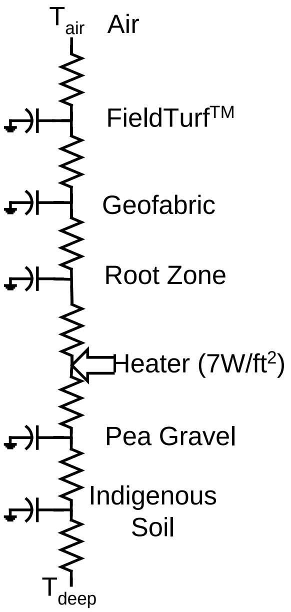


Figure 4: Electrical analog for transient heat transfer, showing incorporation of capacitances for each material.

Baughn [2] discussed the concept of a time constant (and its obvious analog in electrical engineering), and showed that the maximum allowable timestep size (to ensure numerical stability) is subject to the criterion

$$\Delta t < RC_{min} \equiv \left(\frac{C_i}{\sum_j \frac{1}{R_{ij}}} \right)_{min} \tag{7}$$

where the node with the minimum value of RC dictates the maximum allowable timestep size. Again, the use of the electrical analog is readily evident, facilitates student understanding of the equations, and allows for rapid programming in a spreadsheet environment.

Using the appropriate buttons (Figure 1), students reset the temperature at all node points to the "deep" temperatures and then ran the simulation for 24 simulation hours without the heater activated—this proved sufficient time to bring the model to a quasi-steady state (i.e., if run again for the same duration, essentially the same temperature profile would result). The resulting temperature values were copied to the column headed "T_{quasi}" so that whenever subsequent runs with the heater activated were desired, the values in this column could be used as an initial condition.

When either of the appropriate set points for the two system control variables (field surface temperature and maximum allowable heater temperature) was exceeded, the heater would be switched off. Again, the objective was to reach a desired field surface temperature for a 4-hour time period on game day, so that there was some trial-and-error to this technique. Students could vary the amount of simulation time the program was run, and also the screen refresh rate (via a VBA slide bar on the worksheet). Consequently, most found it best to run the program very quickly (starting at midnight) for 12 hours, specify a much slower execution speed, run the program for four more hours, and so on. The program calculated the amount of time the heater was on or off. Specifically, students had to calculate how far in advance of game time the heater control system would need to be activated to support the field temperature criterion, and what settings were appropriate for the control system. Additionally, they had to research local utility prices and determine the cost of system operation. Further, for back-to-back home games they were required to investigate whether it was better to turn off the heater system after the first game and reactivate it prior to the second, or to leave the system on the entire week in between games.

In a separate exercise, students were also required to change the timestep size to twice the critical value, and to run the model in 1-hour increments, save the resultant temperatures at the end of each hour, and plot these temperature distributions in order to display the concomitant instability Figure 5). They were also required to comment on the nature of the instability (point of origin, magnitude, speed of propagation, etc.). Most students were able to quickly ascertain that the point of origin was at the point where the RC_{min} was produced in the cross section (this occurred at the geofabric).

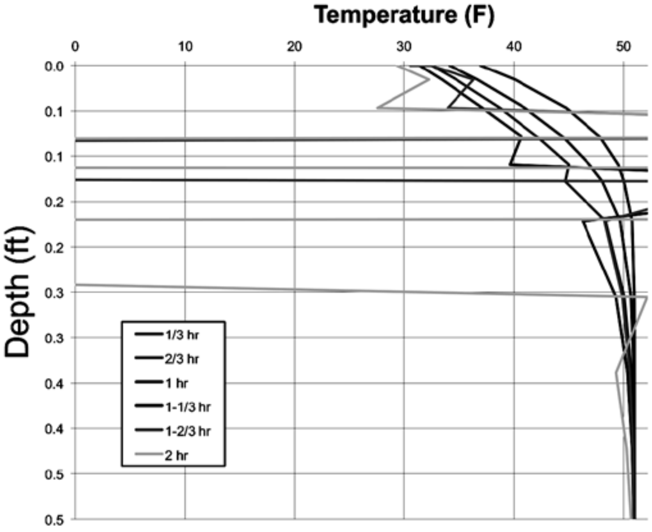


Figure 5: Development of numerical instability associated with excessive time step size in Gauss-Seidel iteration.

The transient heat transfer project was not due for another week after the numerical block completed in order to allow enough time to ask questions.

The VBA code associated with the Excel™ file may be readily modified to accommodate many different situations including variation of heater power, sunrise or sunset, peak insolation value, ambient temperature profile, and changes to convection coefficient which may be linked to a typical wind speed profile [6], etc.

Mini Design Project Possibilities

Finally, students were required to choose (or devise) a mini-design project where their model could be productively employed; if a topic was suggested, it was required that students propose it to the instructor so that it could be verified as a legitimate project, and that it not be too simple (or too difficult!). Each project required some sort of financial or duty cycle analysis. Suggested possibilities included:

A. Changing the
  1. Heater power setting
  2. Temperature of the heater shut-off setting
  3. Location of the heater
  4. Thickness of various layers (pea gravel, soil, etc.)

B. How would the system perform with
  1. A significant "cold snap"
  2. Exceptionally high wind
  3. Exceptionally dense cloud cover

C. Consider performance in a different seasonal environment, altering the ambient temperature and insolation as functions of time

D. Alter the distribution of the electric heater
  1. Change the number of cells "distributing" the heat
  2. Instead of an equal amount of heat input in each of multiple cells, taper it off as distance from the nominal heater location increases

E. Alter the size/number of FDM elements

F. Investigate the impact of other game times (i.e., ESPN or CBS College Sports television time requirements)

Student Feedback

Following completion of the numerical heat transfer block, the 27 students were queried regarding the efficacy of the teaching method, homework, and projects in presenting the material; the results of the written survey are presented in Table 1. When shown the text which contained the more traditional teaching approach for the finite difference method as applied to heat transfer [7], the students were overwhelmingly in favor of the electrical analogy method described herein. Additionally, they found the format, difficulty, and timing of the assigned homework and projects to be very useful, and admitted that the (almost) daily mandatory submission of homeworks allowed them to remain abreast of the material. The students found the exposure to computer modeling techniques to be useful, although many did not believe that it would be directly applicable to the capstone projects in which they were involved (Instructor experience, however, has shown that this tool has been used to great benefit in many of the various capstone projects. Specific instances include braking analyses of intercollegiate competition cars (FSAE/Baja), analysis of unmanned aeronautical vehicle motor heat dissipation devices, rocket igniter heat transfer analyses, and rocket nose cone heat transfer analysis. In each of these instances, a seemingly intractable problem that could not be solved as a classic "textbook" problem was now rendered feasible with a series of valid assumptions.). Finally, the students felt that the project was very worthwhile in acting as its own "capstone," since it integrated all three modes of heat transfer in a single assignment. Informal, anecdotal feedback from the students indicated that, once mastered, the final design project proved to be a valuable tool which allowed them to draw conclusions about how changing various parameters would affect system operation.

While results prior to 2007 were not readily available, average scores for the afternoon heat transfer portion of the mechanical engineering version of the Fundamentals of Engineering exam showed that USAFA mechanical engineering majors exceeded the national average by 36 (only 1 student), 7, and 18% in 2007, 2008, and 2009, respectively. While the specific impact on these data which may be attributed to projects such as those described herein, these projects serve as integrating "capstone" experiences which cause students to reflect on the entire heat transfer curriculum, and undoubtedly aid in material retention.

Recommendations for Implementation

After several years of implementing projects such as the one described herein, there are several items which the authors feel are worthy of note:

Table 1: Student Survey Results.

Question	% Agreeing
Use of the electrical analogy made finite difference (FD) heat transfer easier to learn than the classical method	95.8
Homework (HW) was not excessive	75.0
HW was helpful	95.5
I probably wouldn't have done the HW when assigned, had it not been collected daily	69.2
HW was of the appropriate difficulty	79.6
The design portion of the project allowed me to develop and use a computer model	66.7
Numerical heat transfer (NHT) will probably be of no use to me in my capstone	58.3
The NHT block was good at integrating the various modes of heat transfer	95.8

- While one of the principal issues at USAFA was the lack of teaching assistants (or other grading help), should a project of this nature be implemented at a school where such resources are available, students could be required to "start from scratch" (i.e., without a template from which to start), and possibly be given a somewhat longer time period during which to perform the projects.

- The use of commonly-held software at the institution is recommended, preferably one which has a good visual interface (i.e., plotting) capability. While Excel™ seems to be the choice in most instances, this does not obviate the possibility of other software packages that might be readily available and in which the students have experience (e.g., MatLab™, Mathcad™, etc.)

- A gradual build-up in difficulty in introducing this material is considered most appropriate, as is a "standard" analysis required of all students.

- The real benefit of a tool such as the FDM, however, is in its application to transient, "open-ended" problems; consequently, the authors feel it is most beneficial to require all students to conduct some sort of "mini-design" project in conjunction with this block of instruction.

- Several recommendations are offered by Baughn [2] and in this paper regarding potential topics for this sort of FDM project. Regardless of which topic is chosen, it is highly recommended that the electrical analogy be used to the maximum extent possible. It is a common sense approach to what many students view as a daunting topic and provides a concrete link to the presentation most schools use while introducing the three modes of heat transfer. Again, it is a nice way to synthesize and integrate the topics covered in a standard heat transfer curriculum while using "real-world" problems as a vehicle.

Conclusions

Perhaps the best indicator of success of any such instructional program is the enthusiasm and extent with which the material taught is applied throughout other aspects of the curriculum. From the authors' experience, it is "just in time teaching," since there are routinely numerous instances whereby the FDM has been put to practical use in the capstone sequence at USAFA. Student feedback indicates that the gradual increase in difficulty, mandatory daily hand-ins, and continued use of the electrical analogy resulted in good comprehension of a topic which is considered very difficult by many students.

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